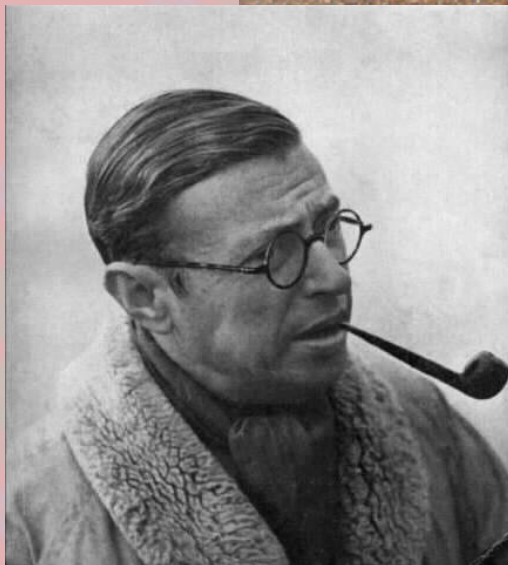


# working with **Graham Ross** 1974-1984



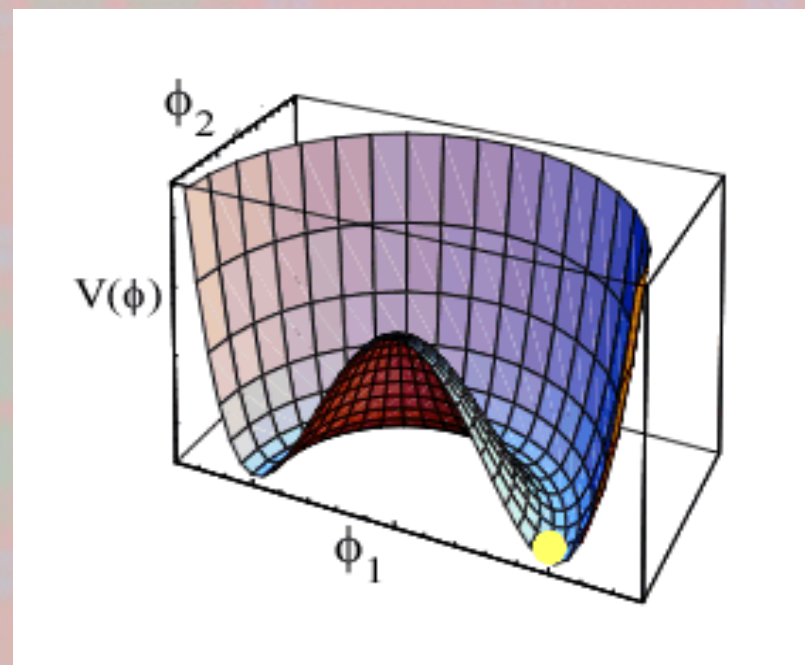
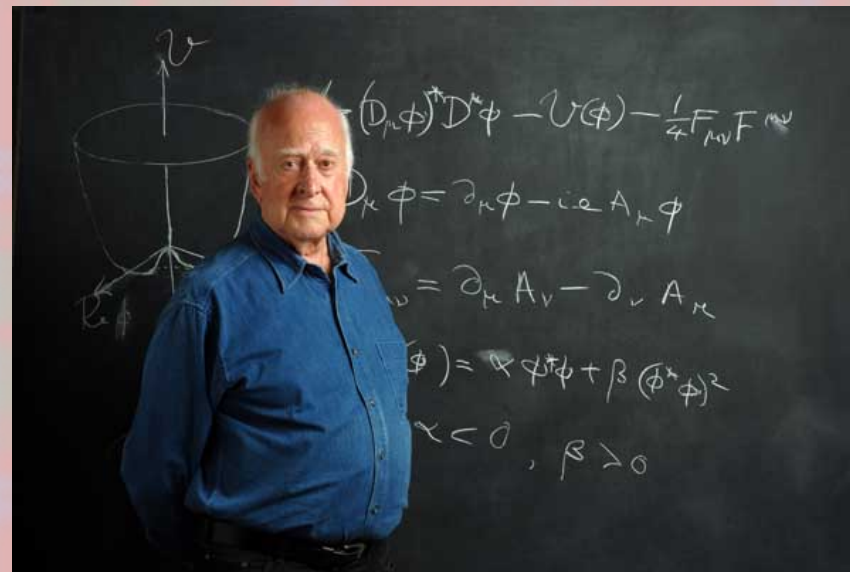
**Les Chemins de la Liberté**

# working with **Graham Ross** 1974-1984

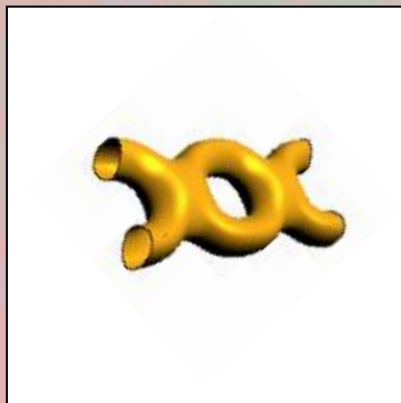
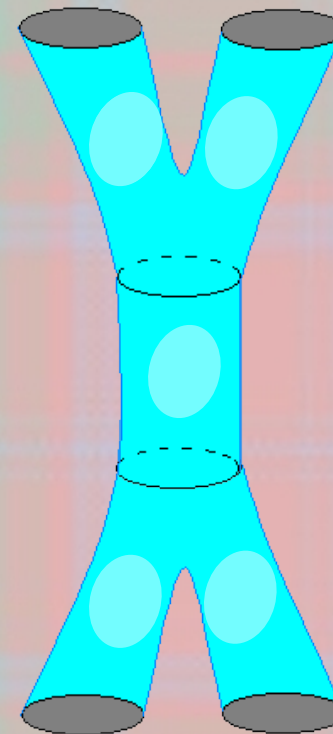
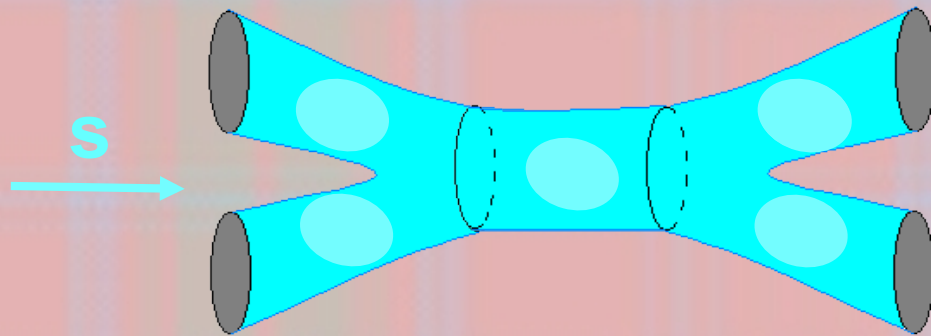


# working with **Graham Ross** 1974-1984

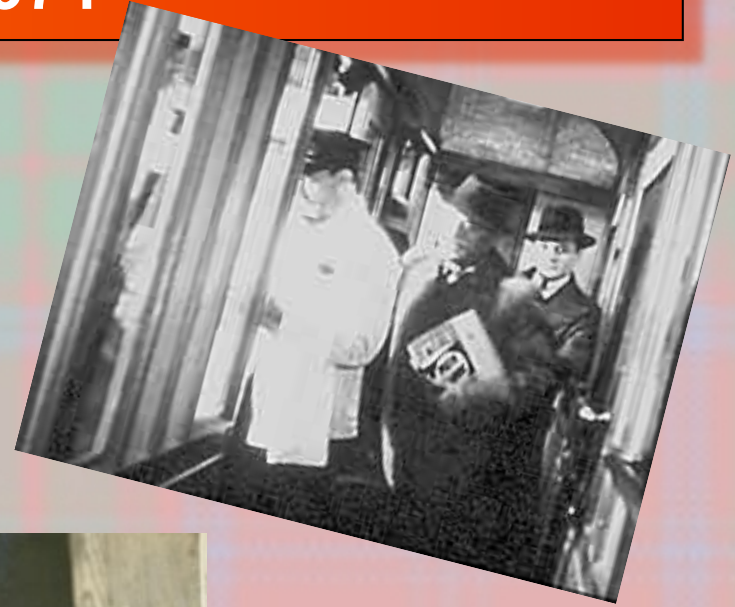




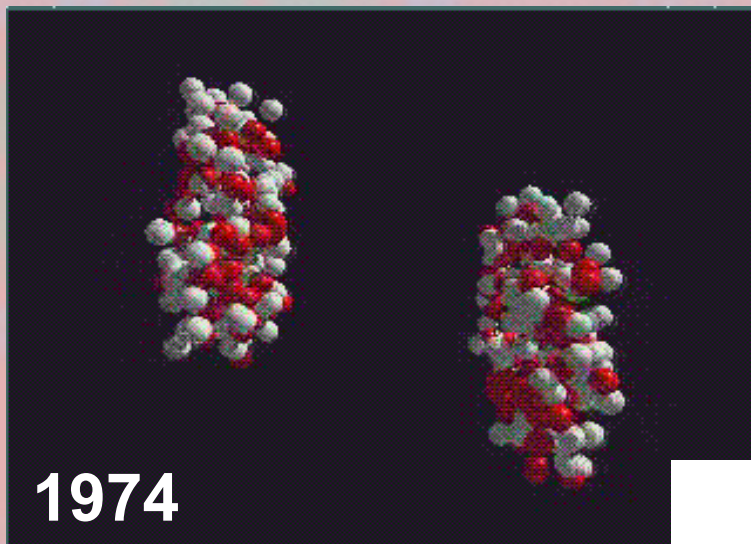
# Duality & Unitarisation



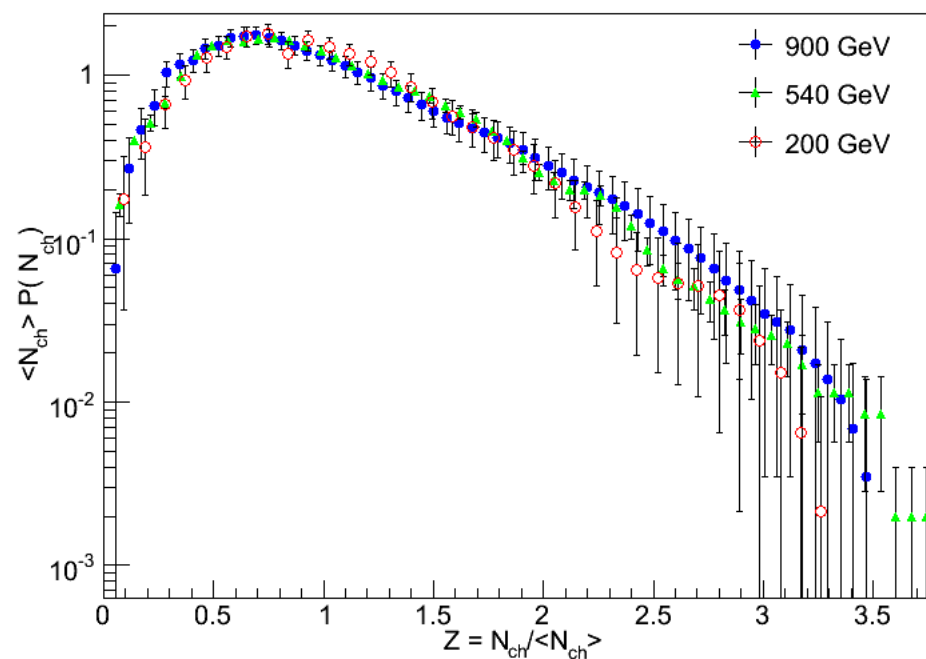
# XVII International Conference on High Energy Physics, London, July 1974



# Footnote in Physics



KNO Scaling



## Footnote in Physic

Nuclear Physics B88 (1975) 237–256  
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### TESTS OF GEOMETRICAL SCALING AND GENERALIZATIONS \*

V. BARGER and J. LUTHE

*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706*

R.J.N. PHILLIPS

*Rutherford Laboratory, Chilton, Didcot, Berkshire, England*

Received 9 December 1974

the prescription

$$\frac{d\sigma/dt(s, t)}{d\sigma/dt(s, 0)} = f(\sigma_t^2 t / \sigma_{el}),$$

with imaginary non-flip amplitudes and  $f$  some universal function, proposed independently as a generalization of GS by Pennington and Ross [12]. For small  $t$  this prescription is almost trivial, since it is well known that  $d\sigma/dt$  is universally exponential here [13]; the interest lies at larger  $t$ -values.

[11] A. Martin, Nucl. Phys. B77 (1974) 226.

[12] M.R. Pennington and G.G. Ross, to be published.

[13] V. Singh and S.M. Roy, Phys. Rev. Letters 24 (1970) 28.

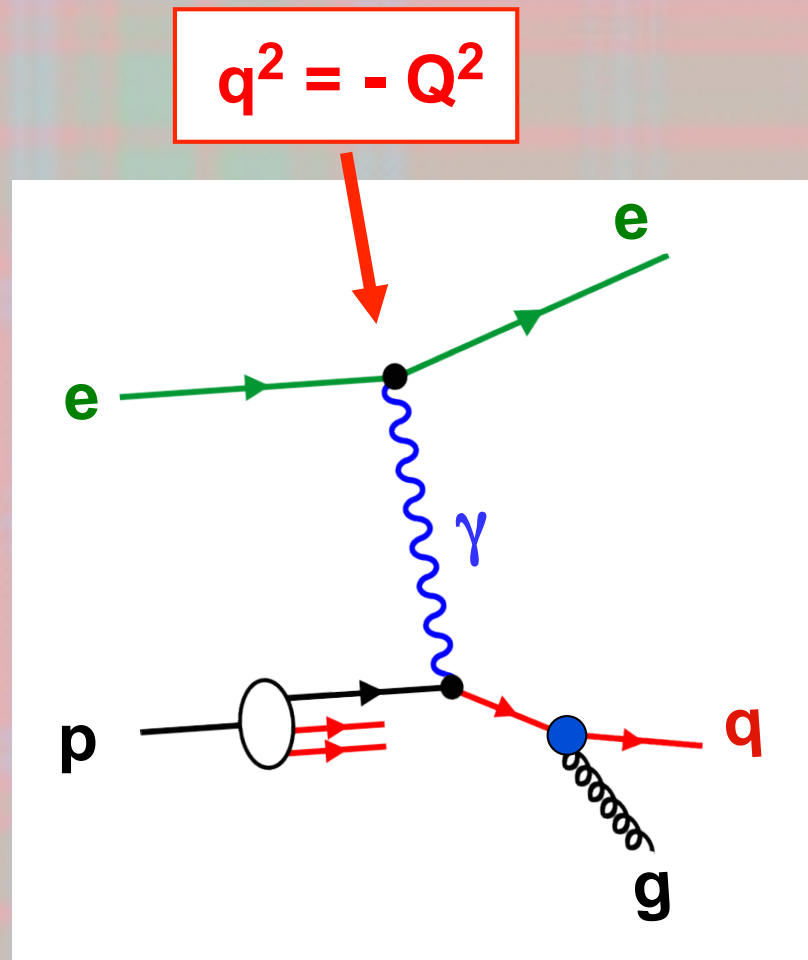
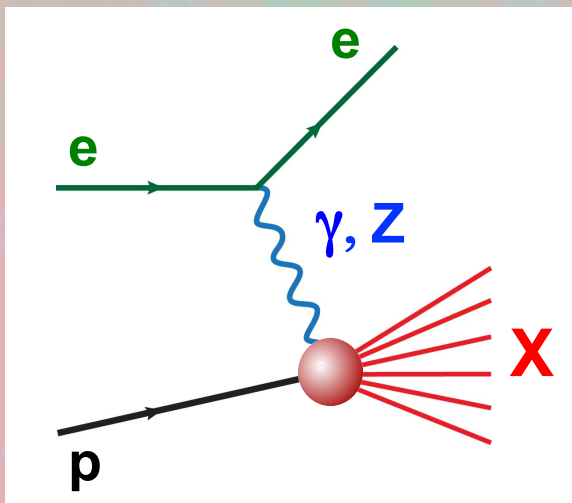




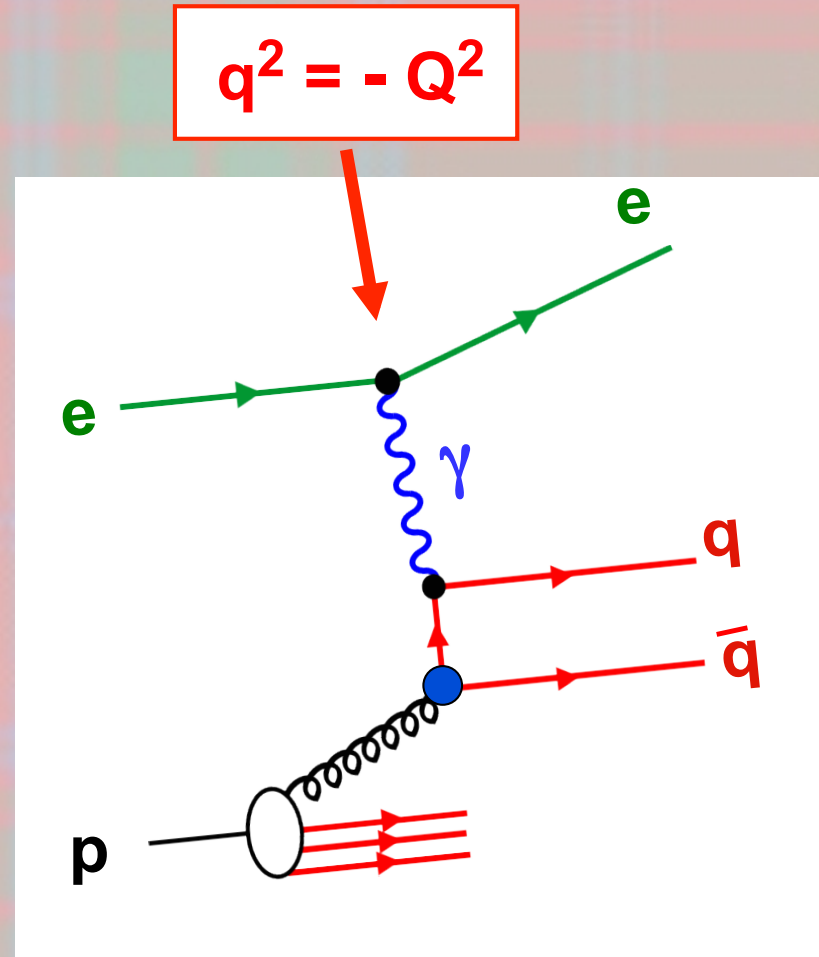
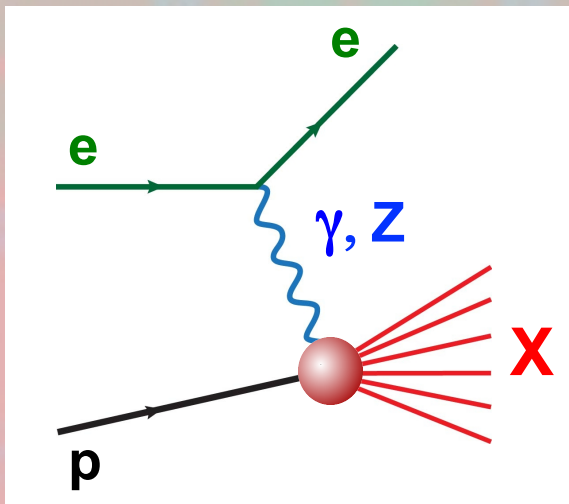
$$\mathcal{L}_{\text{QCD}} = \sum_{q=u,d,s,c,b} \bar{q} (i\gamma_{\mu} D^{\mu} - m_q) q - \frac{1}{4} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}$$

QCD

# DIS, Renormalization Group & pQCD

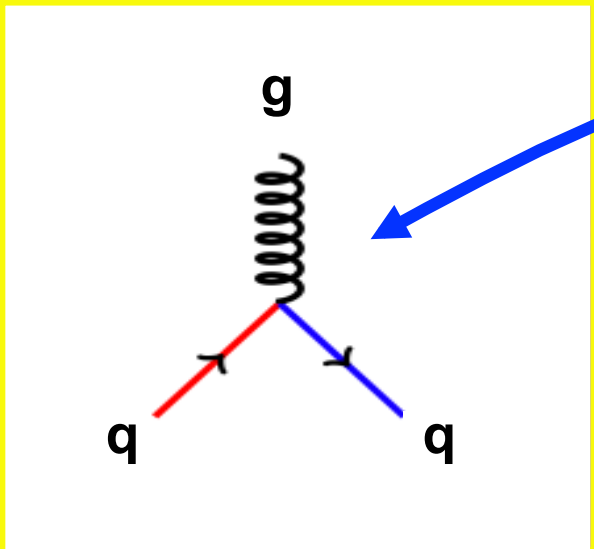
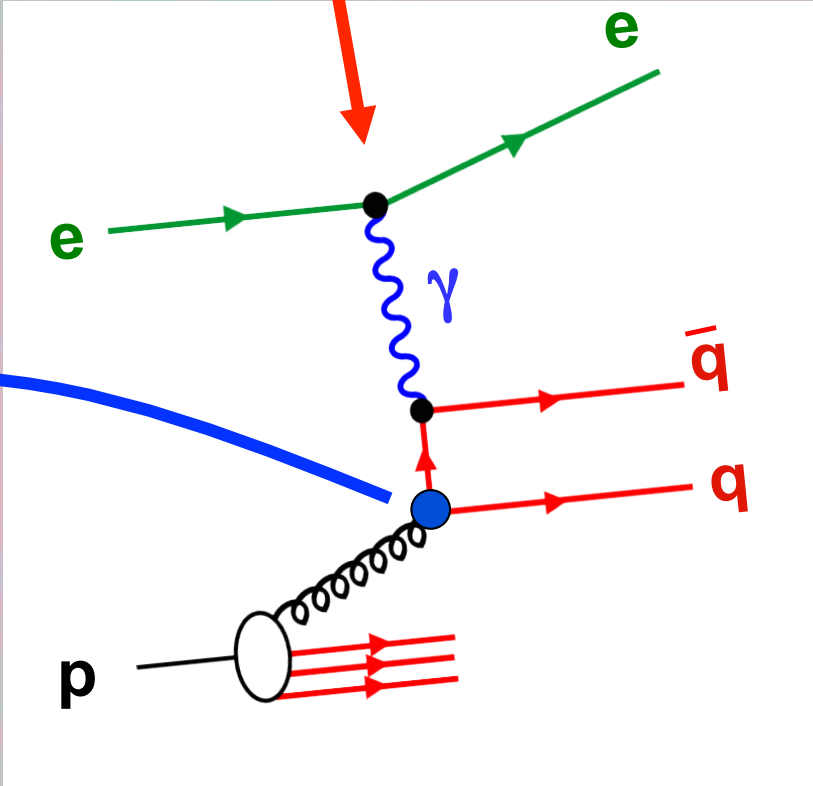


# DIS, Renormalization Group & pQCD

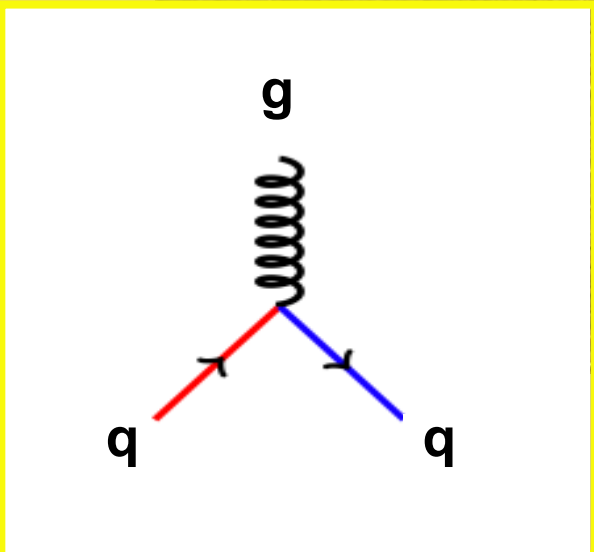


# DIS, Renormalization Group & pQCD

$$q^2 = -Q^2$$



$$\alpha(p_1^2, p_2^2, p_3^2)$$



$$\alpha(p_1^2, p_2^2, p_3^2)$$



Les Chemins de la Liberté

# What can asymptotic freedom say about $e^+e^- \rightarrow \text{hadrons}$ ?



Nuclear Physics B124 (1977) 285–300  
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**WHAT CAN ASYMPTOTIC FREEDOM SAY ABOUT  $e^+e^- \rightarrow \text{HADRONS}$ ? \***

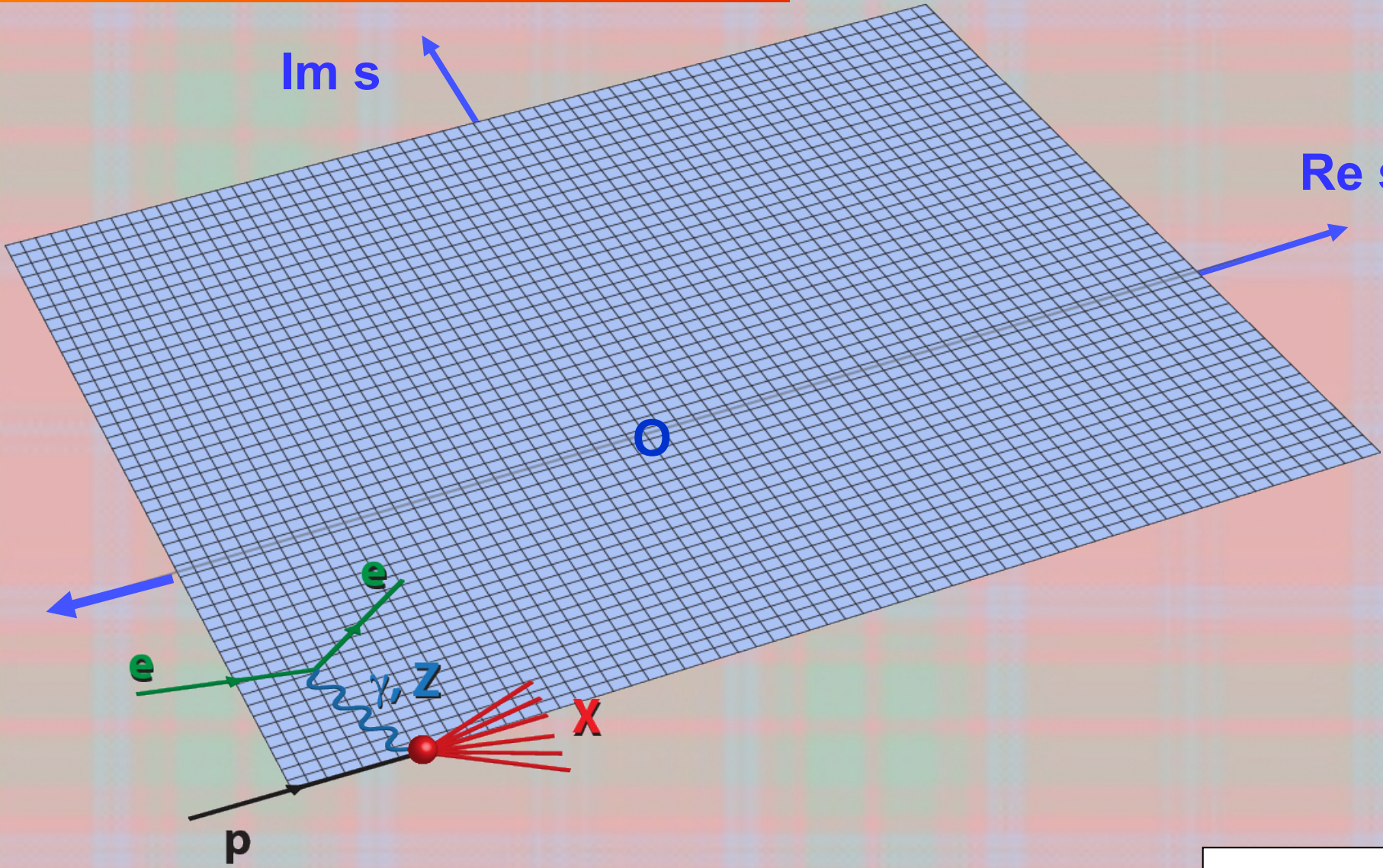
**R.G. MOORHOUSE**  
*University of Glasgow, Glasgow, Scotland*

**M.R. PENNINGTON \*\***  
*University of Durham, Durham, England*

**G.G. ROSS**  
*California Institute of Technology, Pasadena, California 91125*

Received 25 January 1977

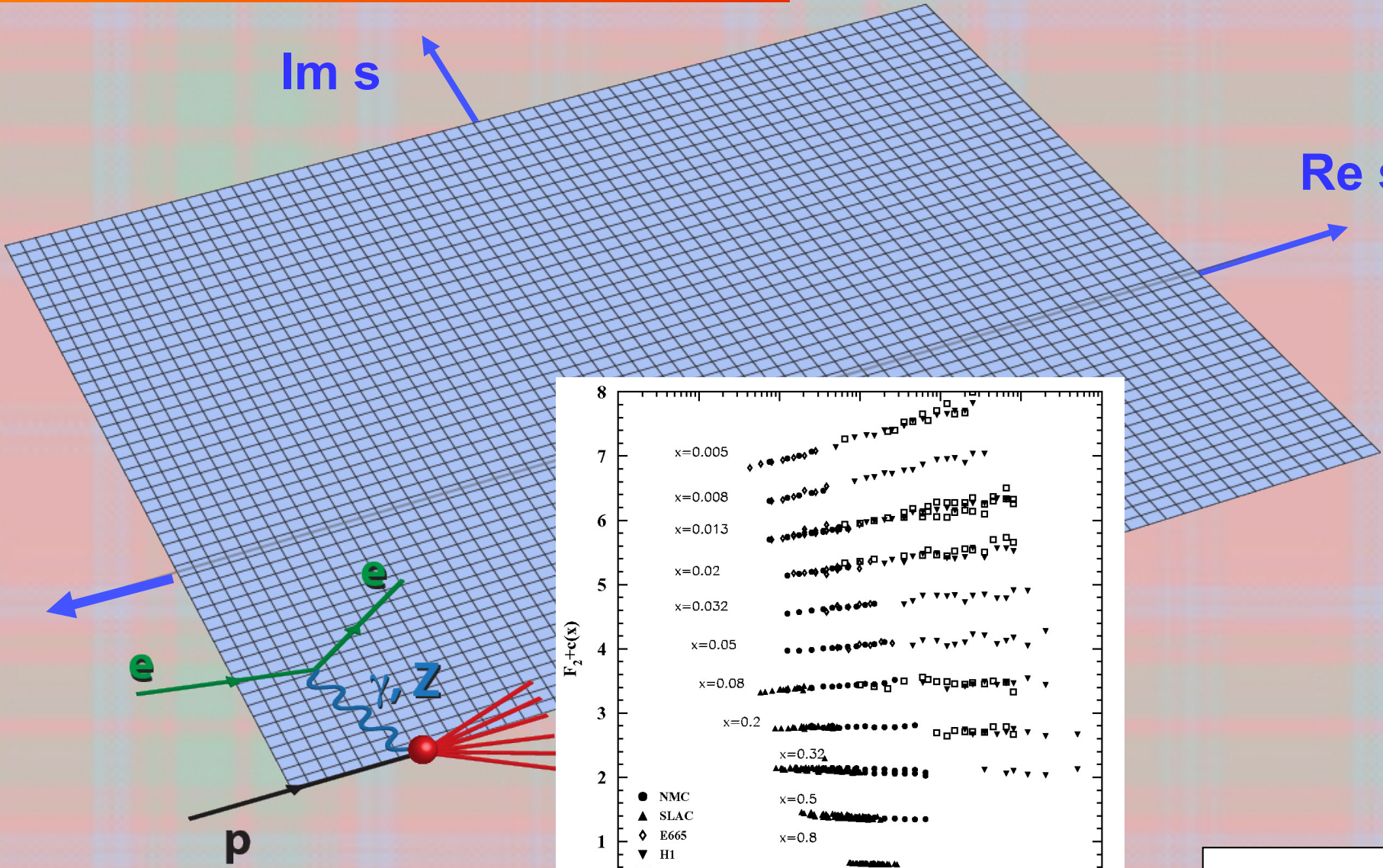
# Where does pQCD apply?



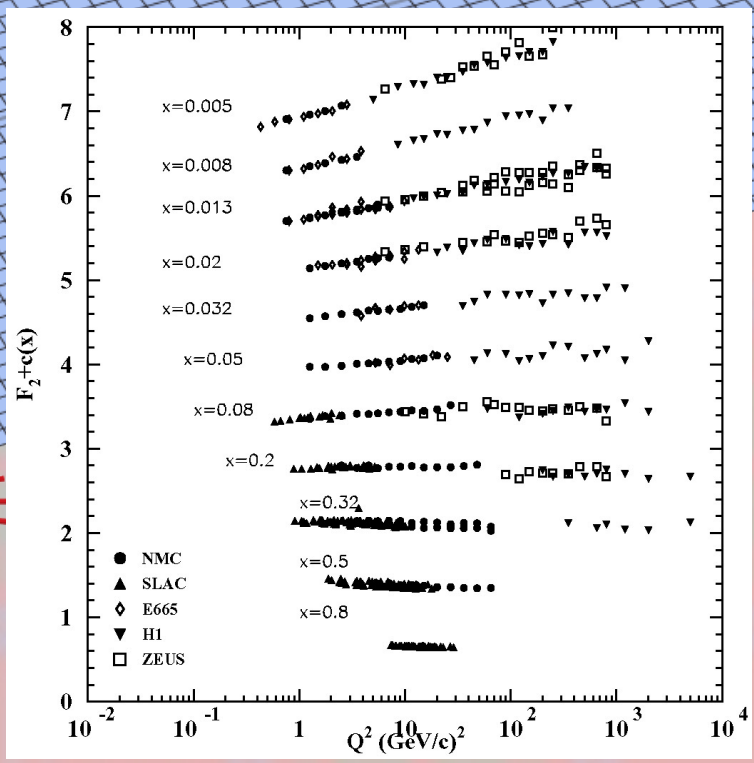
deep Euclidean

$$s = q^2$$

# Where does pQCD apply?



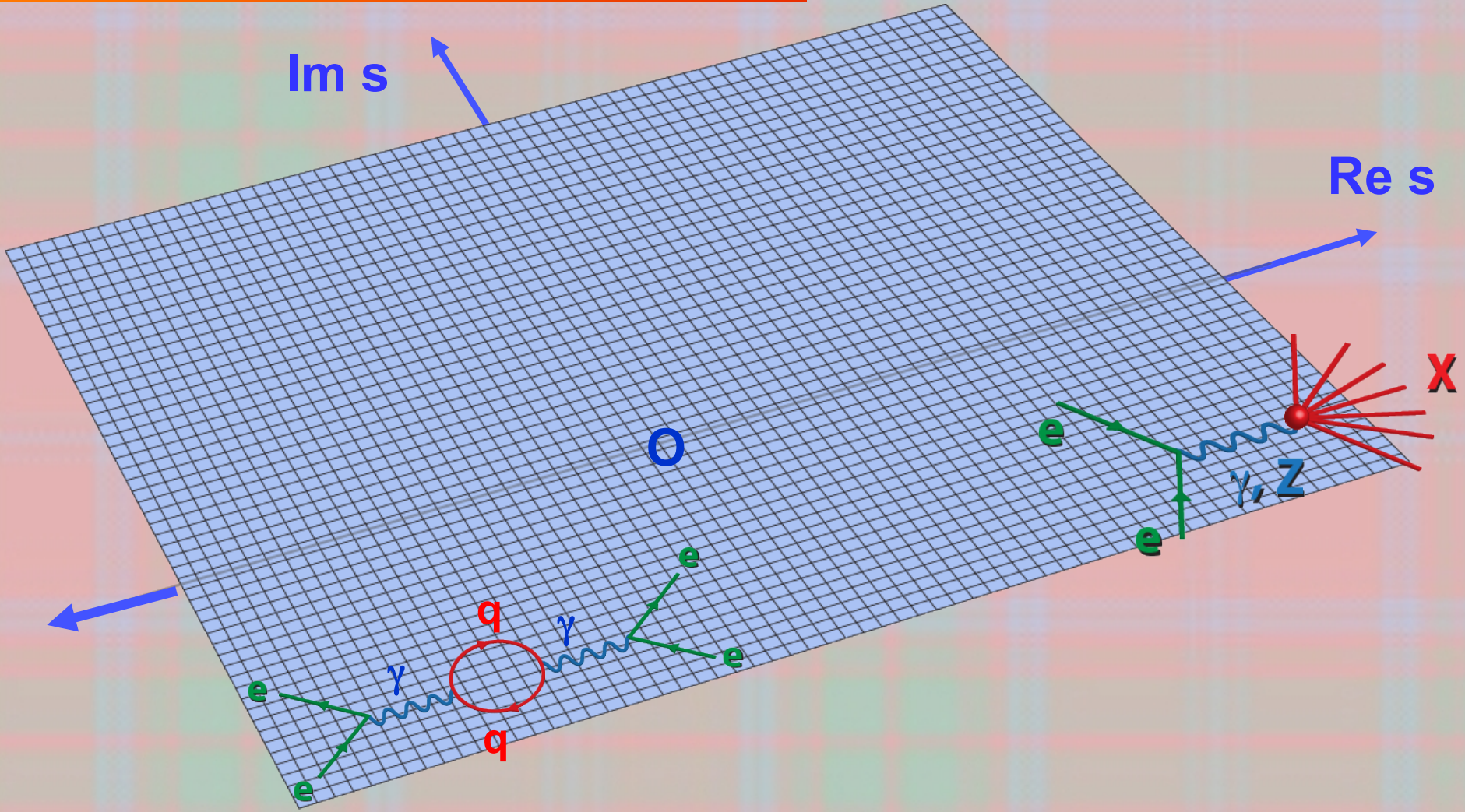
deep Euclidean



$$s = q^2$$



# Where does pQCD apply?

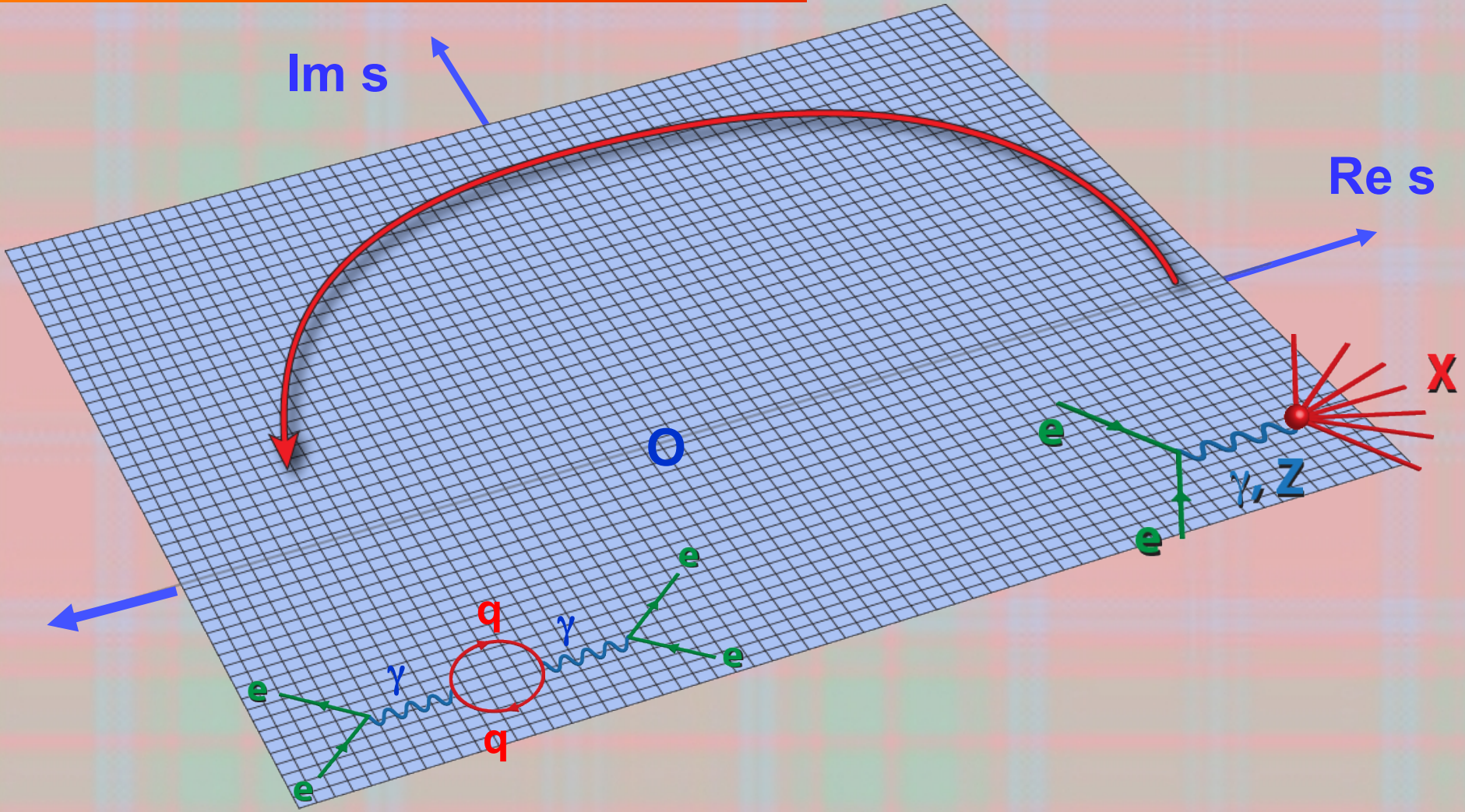


deep Euclidean

de Rujula, Georgi

$$s = q^2$$

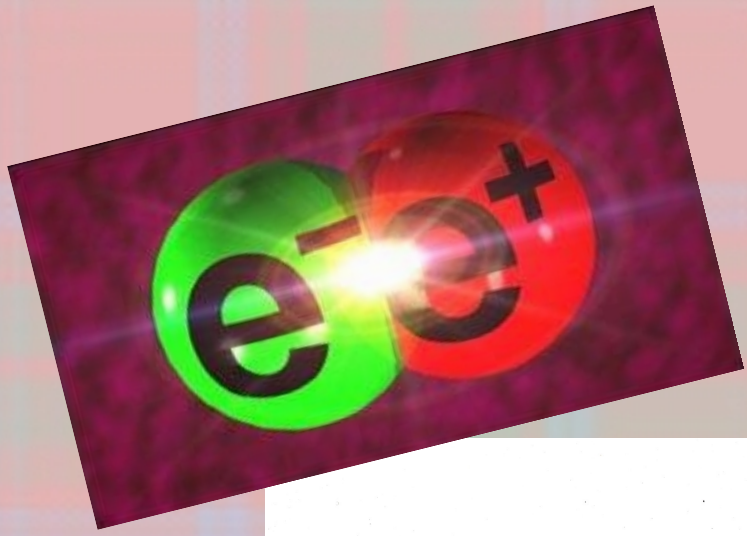
# Where does pQCD apply?



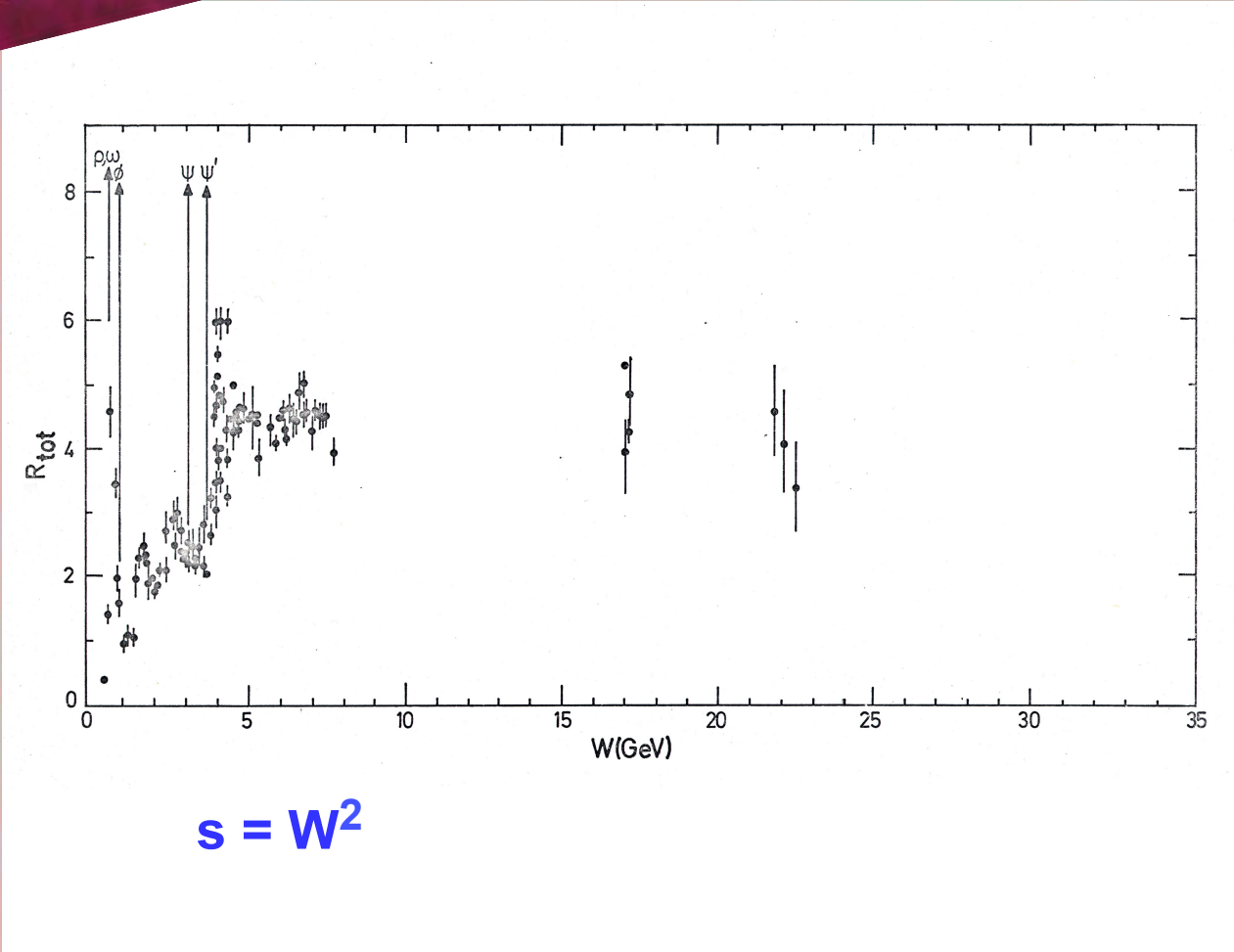
deep Euclidean

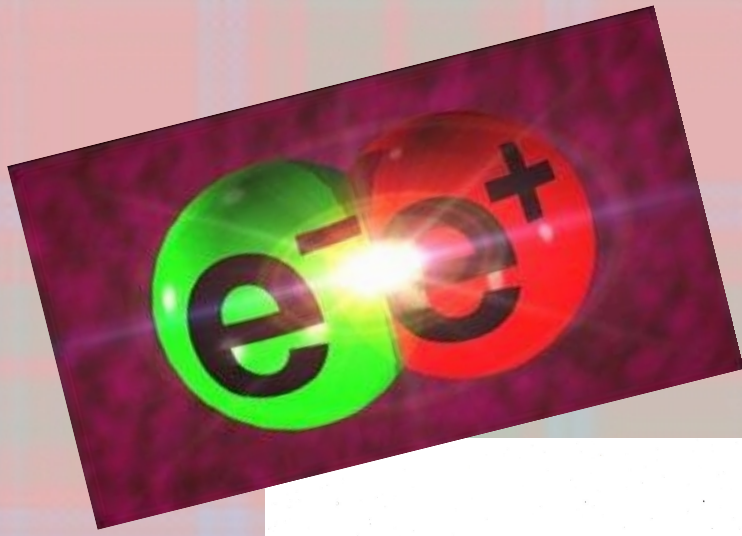
de Rujula, Georgi

$$s = q^2$$

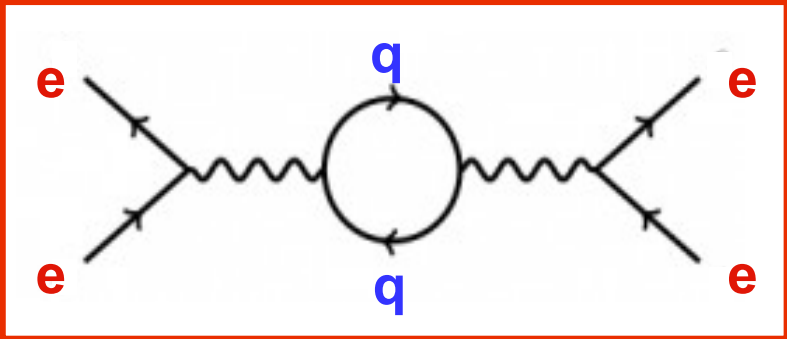
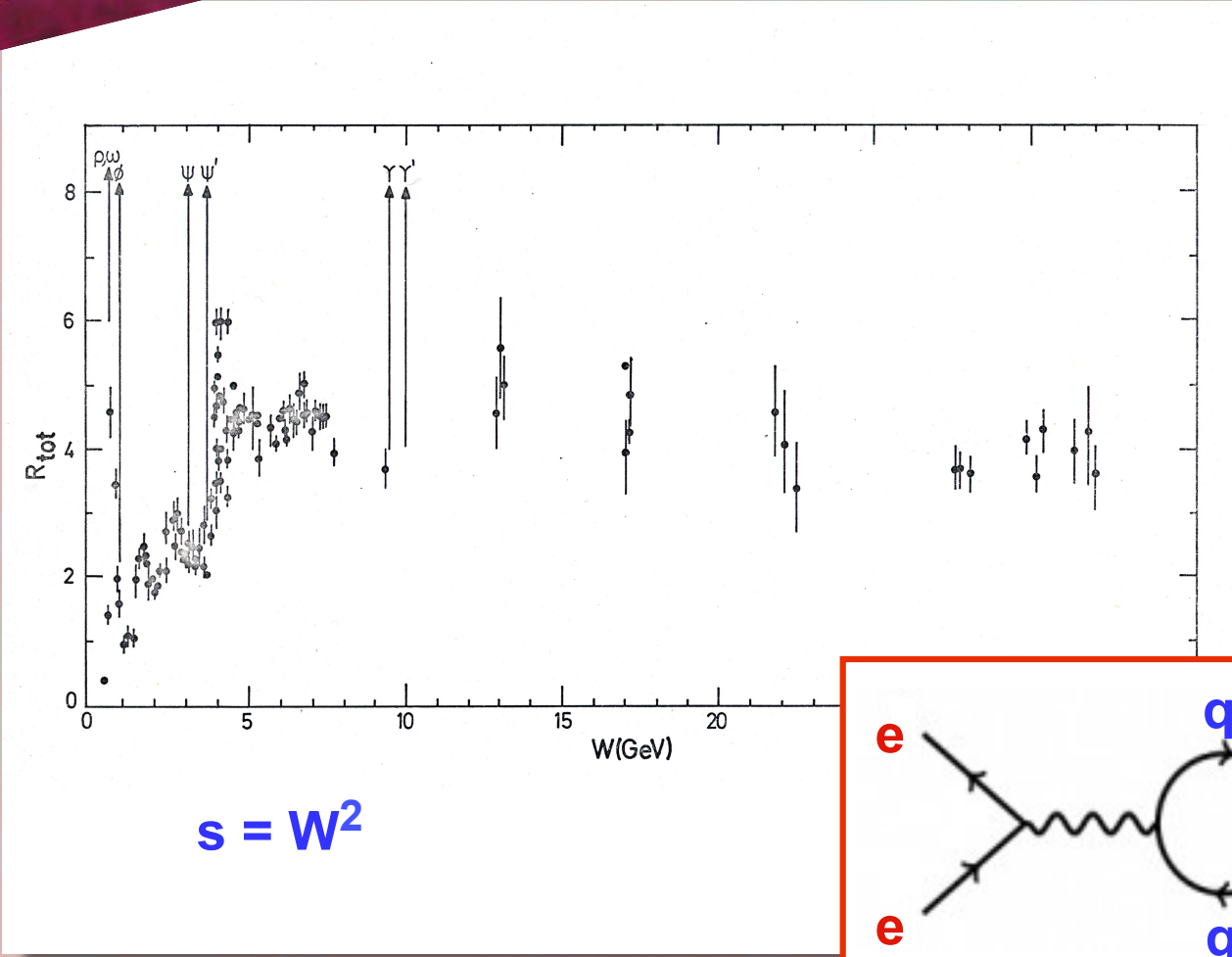


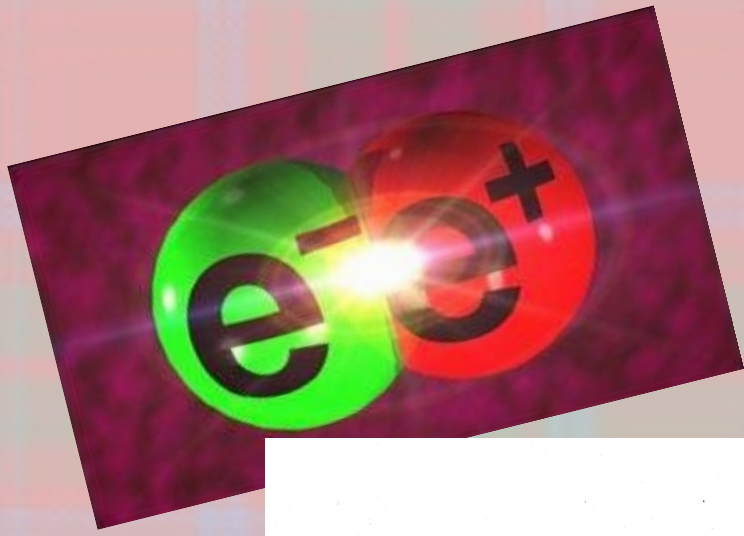
$$R(s) \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



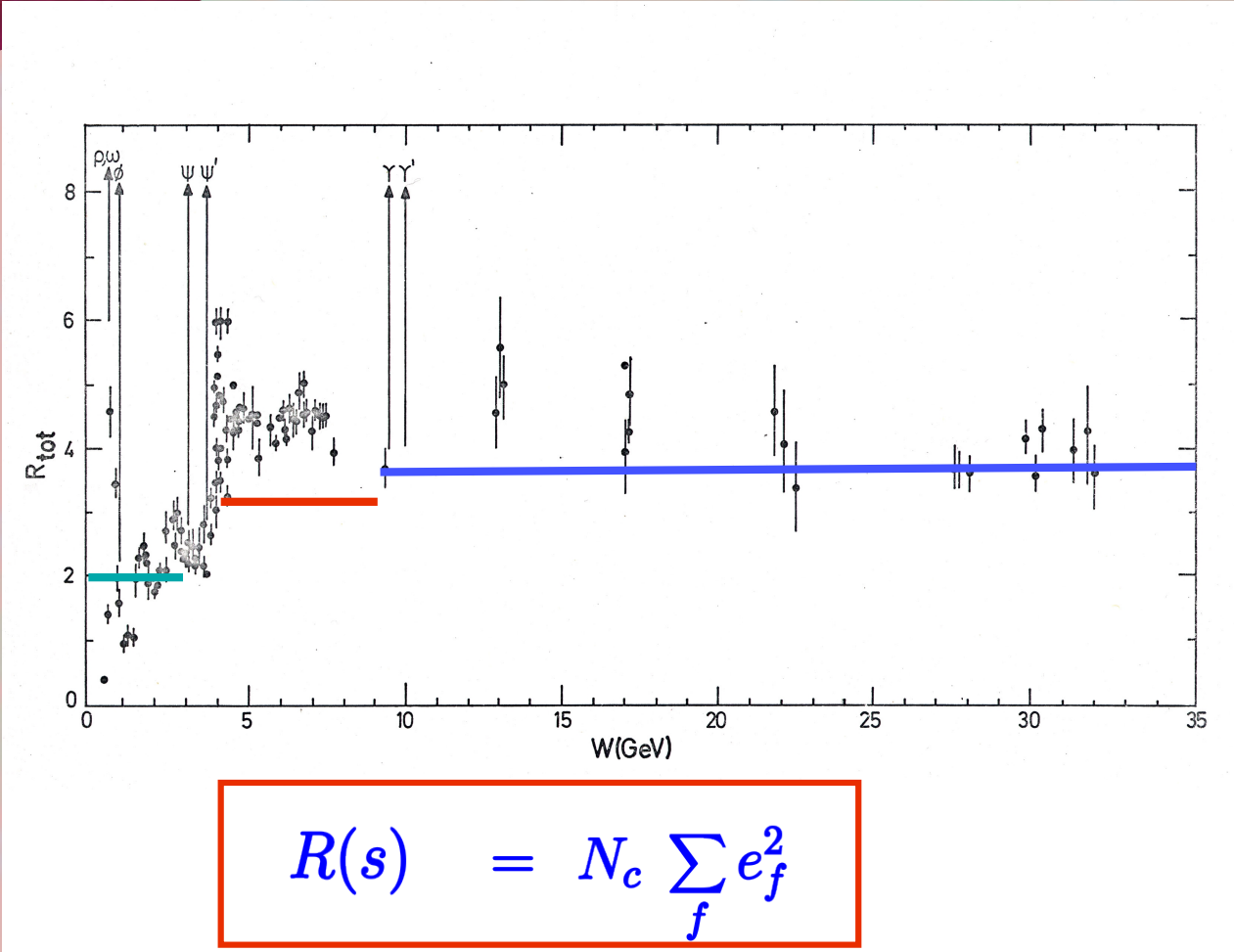


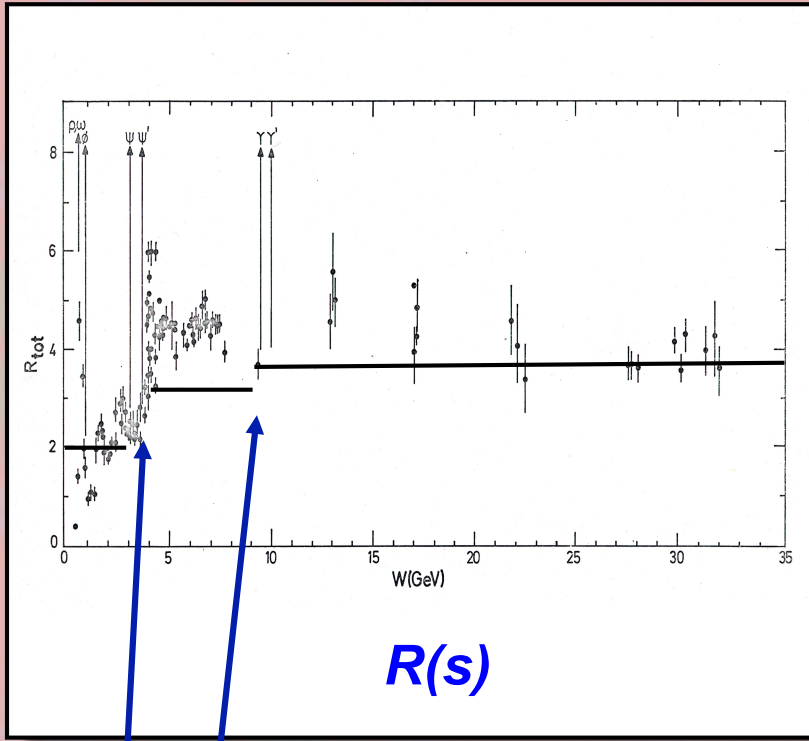
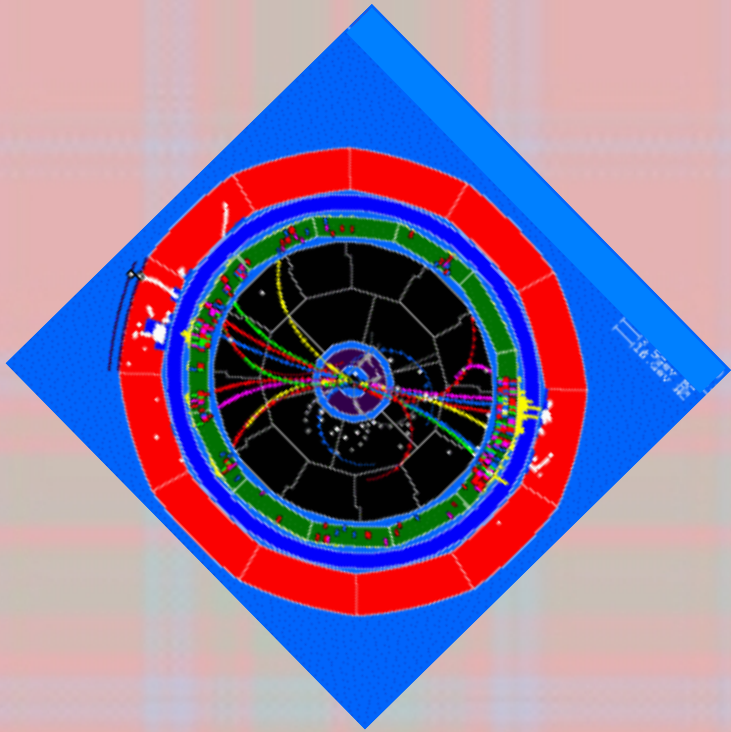
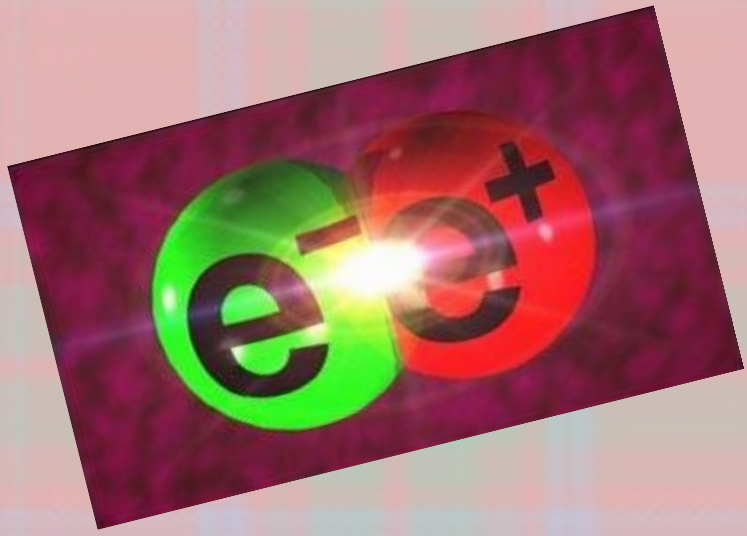
$$R(s) \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$






$$R(s) \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

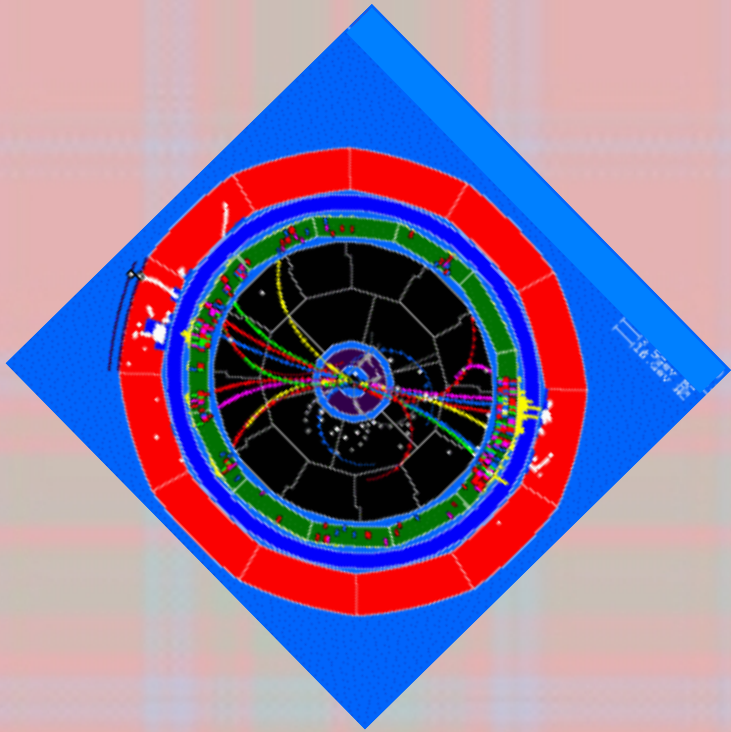
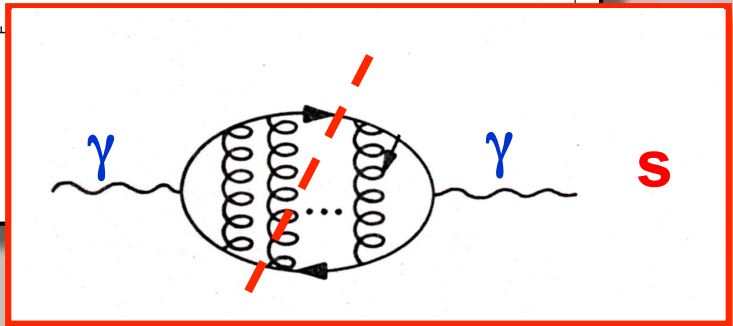
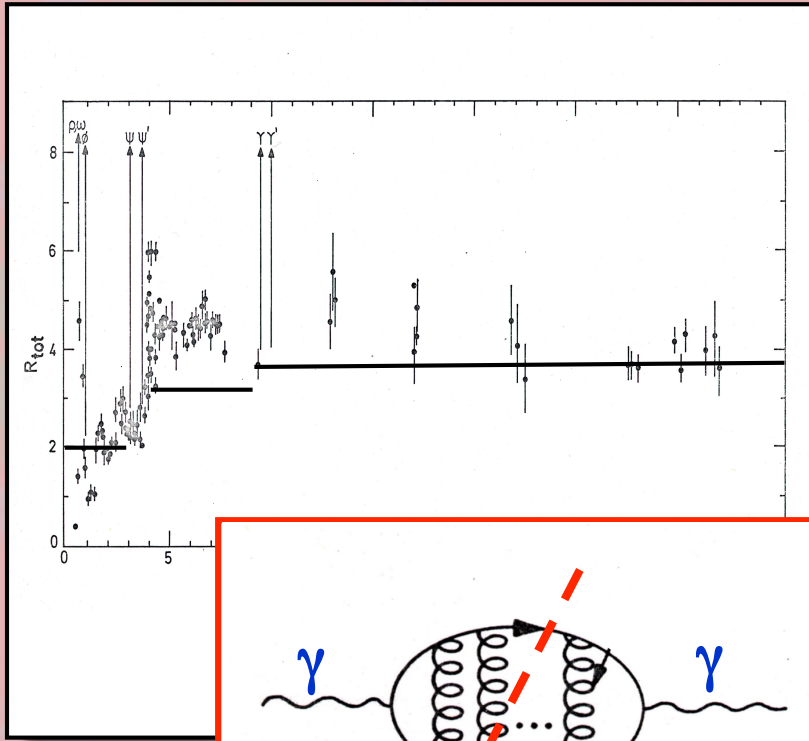
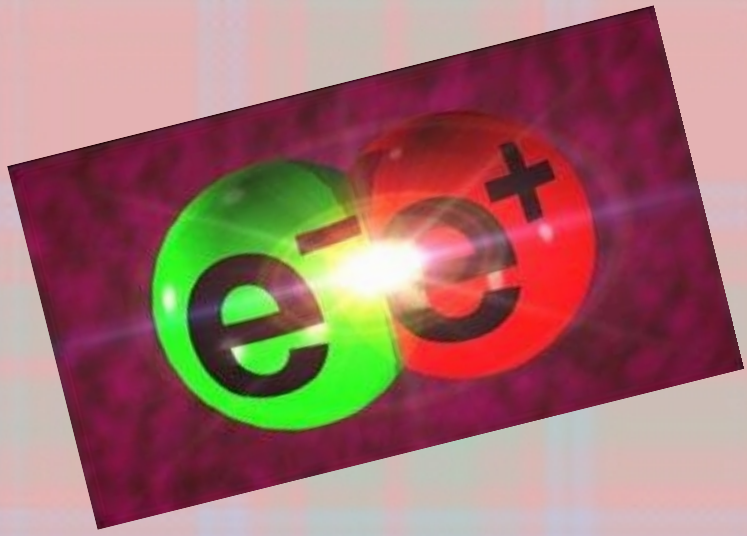





$R(s)$



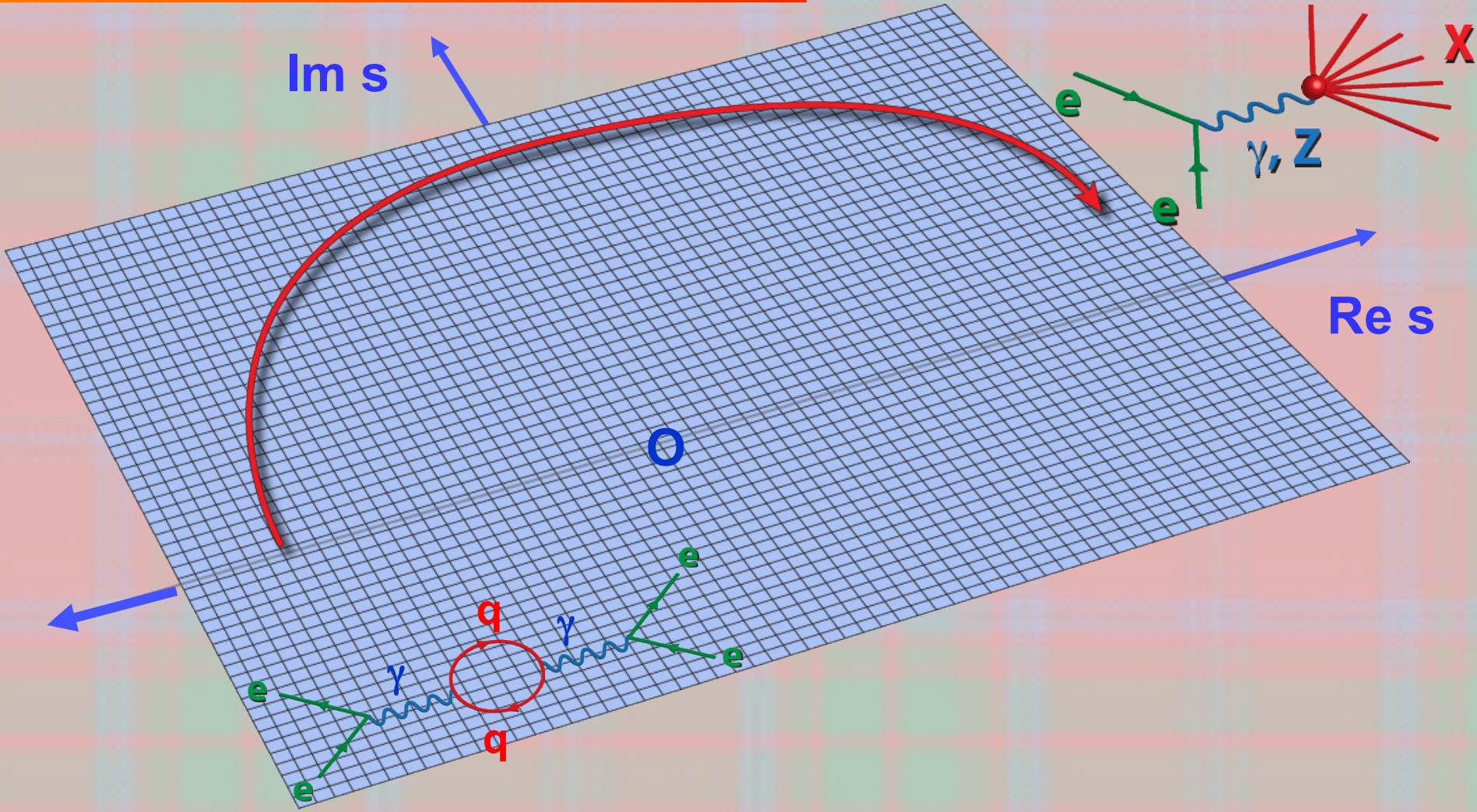
$$S_F(p) = \frac{\mathcal{F}(p)}{\not{p} - \mathcal{M}(p)}$$





$$S_F(p) = \frac{\mathcal{F}(p)}{\not{p} - \mathcal{M}(p)}$$

# Where does pQCD apply?

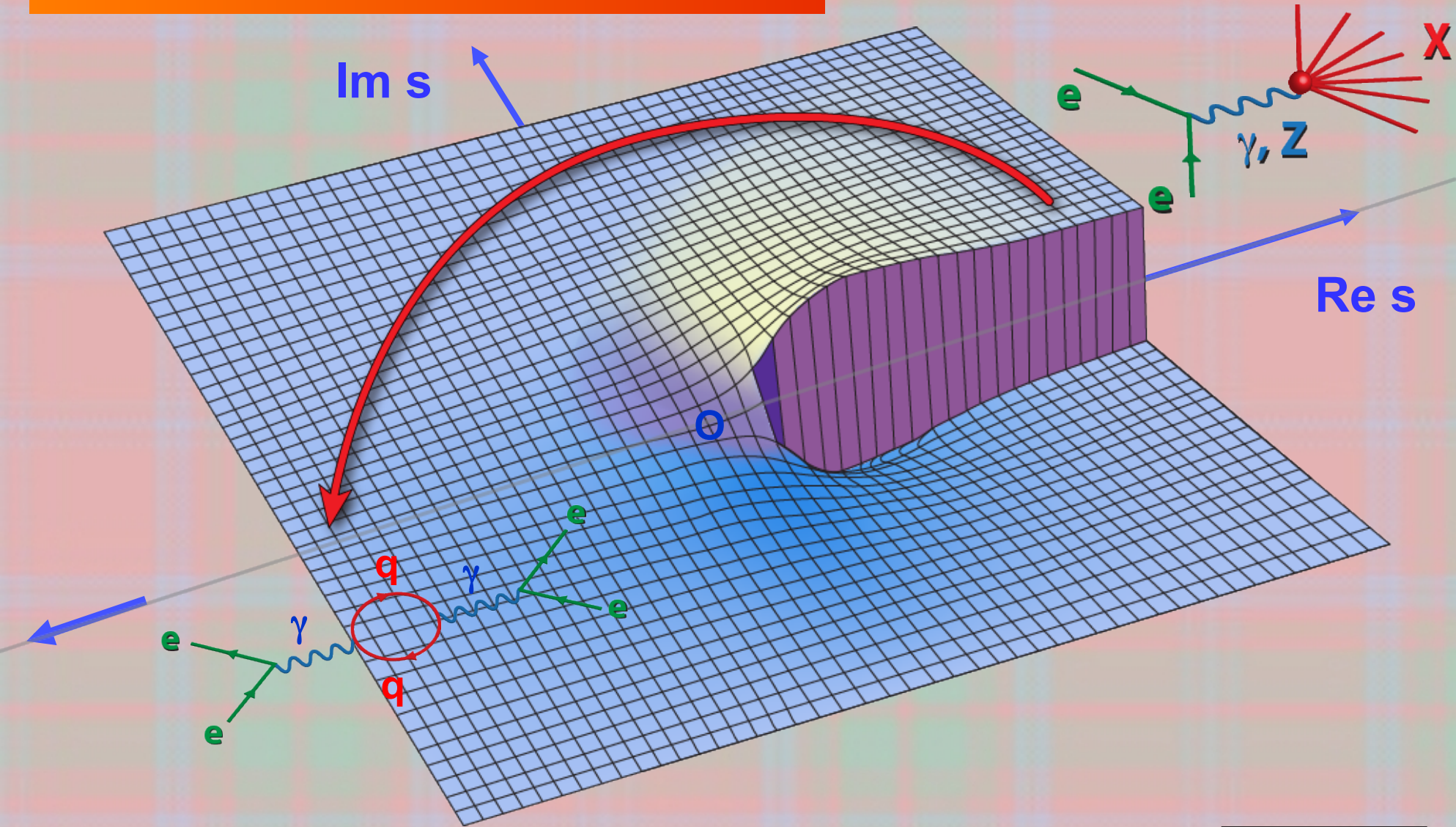


deep Euclidean

$$s = q^2$$



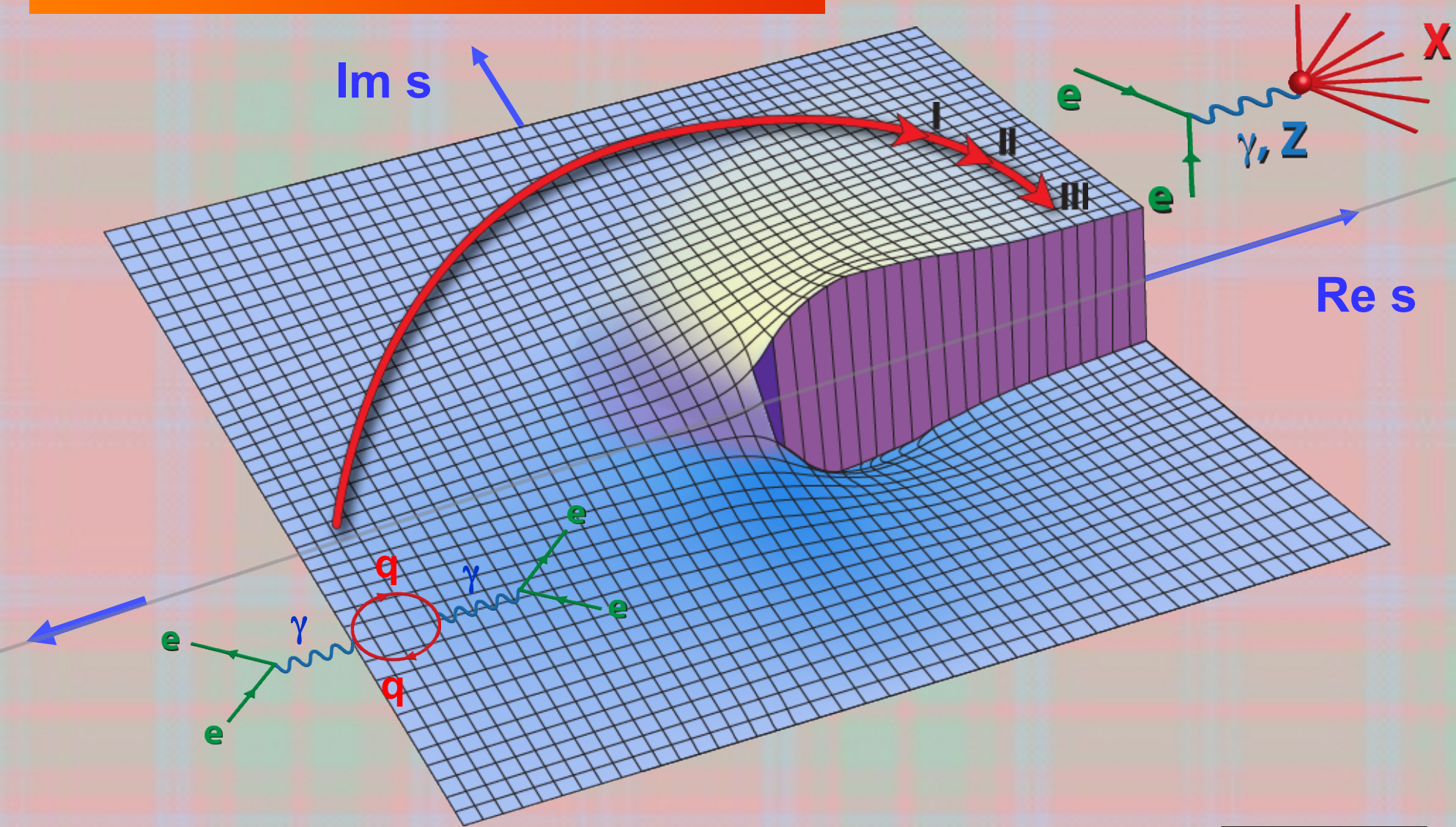
# Where does pQCD apply?



deep Euclidean

$$s = q^2$$

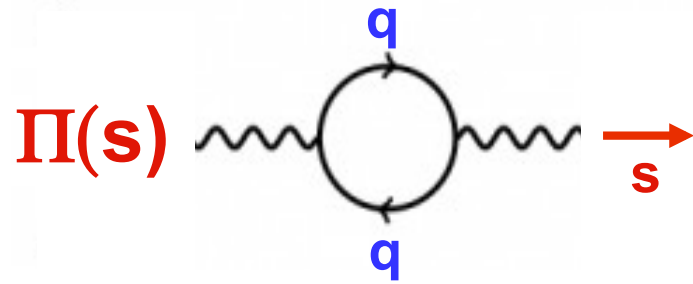
# Where does pQCD apply?



deep Euclidean

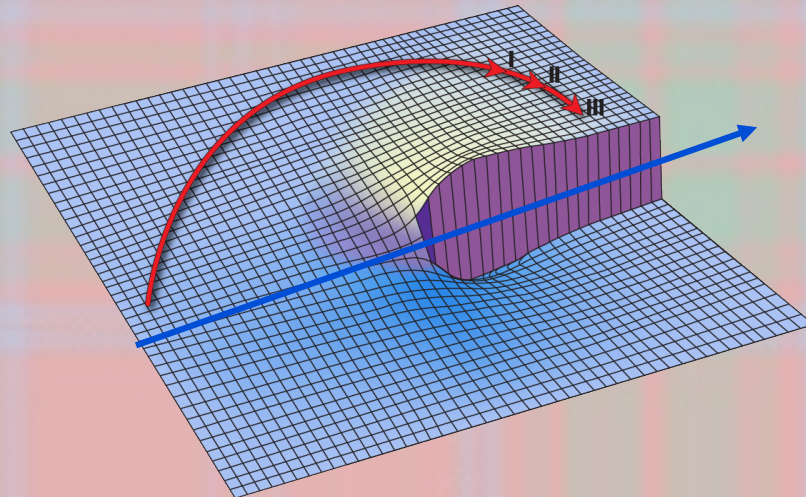
$$s = q^2$$

# Adler $\mathcal{D}$ - function



$$\frac{1}{12\pi^3} \mathcal{D}(s) \equiv s \frac{\partial}{\partial s} \Pi(s)$$

$$\mathcal{D}\left(\frac{s}{\mu^2}, \alpha(\mu^2)\right) = \mathcal{D}(1, \alpha(s)) = \sum_{c,f} e_f^2 \left[ 1 + \frac{\alpha(s)}{\pi} + \mathcal{O}(\alpha^2) \right]$$

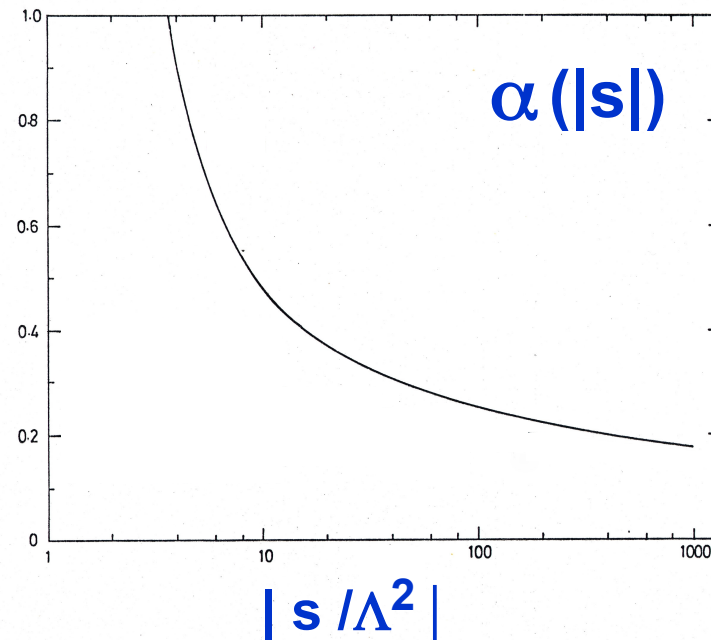


# running coupling

$$\mu^2 < 0$$

$$\alpha(s) = \frac{\alpha(\mu^2)}{\left[1 + \frac{\beta_0}{4\pi} \alpha(\mu^2) \ln \frac{s}{\mu^2}\right]}$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$$



# running coupling

$$\alpha(s) = \frac{\alpha(\mu^2)}{\left[1 + \frac{\beta_0}{4\pi} \alpha(\mu^2) \ln \frac{s}{\mu^2}\right]}$$

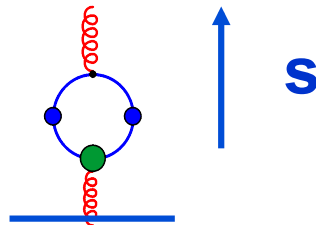
$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$$

timelike  $s = q^2$

$$s > 0$$

$$\mu^2 < 0$$

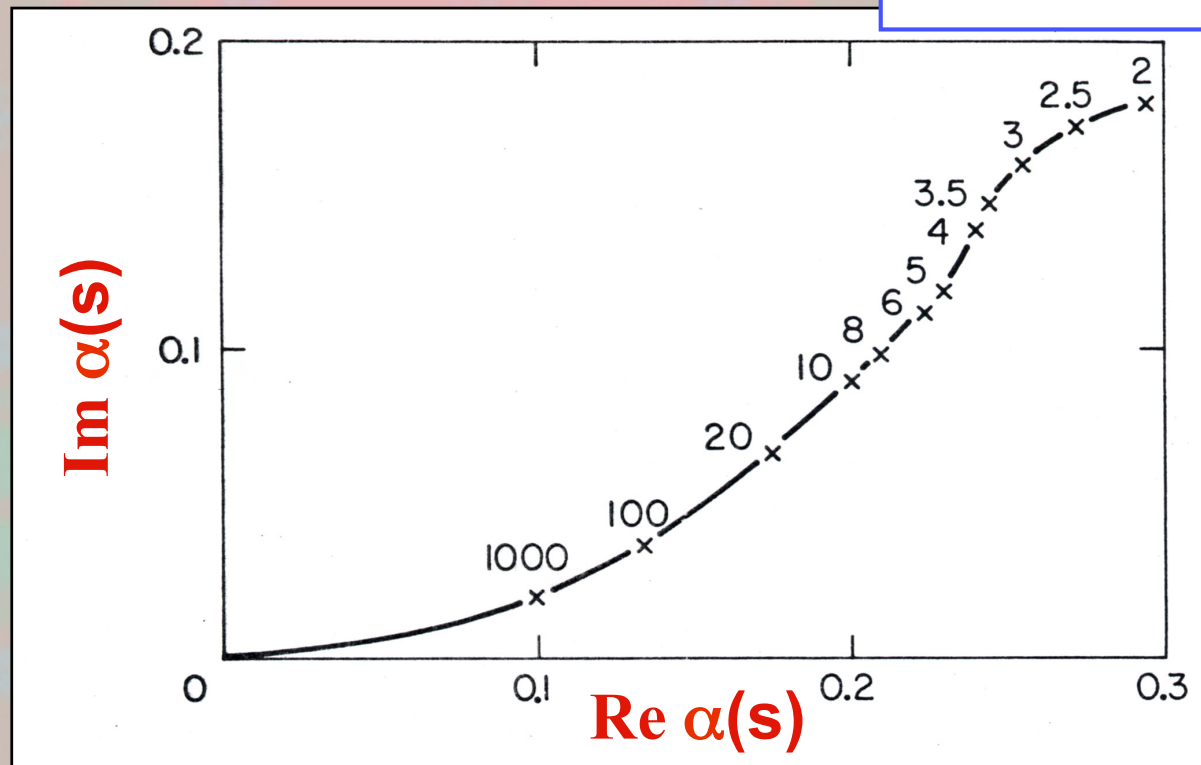
$$\ln \left( \frac{s}{-\mu^2} \right) + i\pi$$



# running coupling

$$\alpha(s) = \frac{\alpha(\mu^2)}{\left[1 + \frac{\beta_0}{4\pi} \alpha(\mu^2) \ln \frac{s}{\mu^2}\right]}$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$$



$$s > 0$$

$$\mu^2 < 0$$

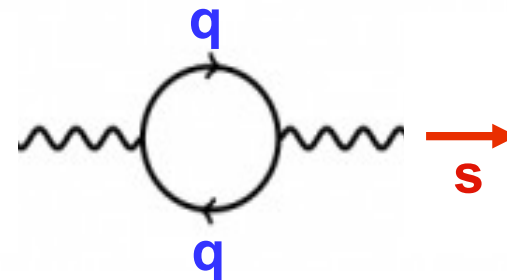
$$S_F^{-1}(p, m) \Big|_{p^2=m^2} = \not{p} - m \quad \text{defining mass at pole}$$

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta(g(\mu^2), m/\mu) \frac{\partial}{\partial g} + \sum_i \gamma_i(g(\mu^2), m/\mu) \right] \mathcal{D} = 0$$

$$\mathcal{D} \left( \frac{s}{\mu^2}, \frac{m^2}{\mu^2}, \alpha(\mu^2, m^2) \right) = \mathcal{D} \left( 1, \frac{m^2}{s}, \alpha(s, m^2) \right)$$

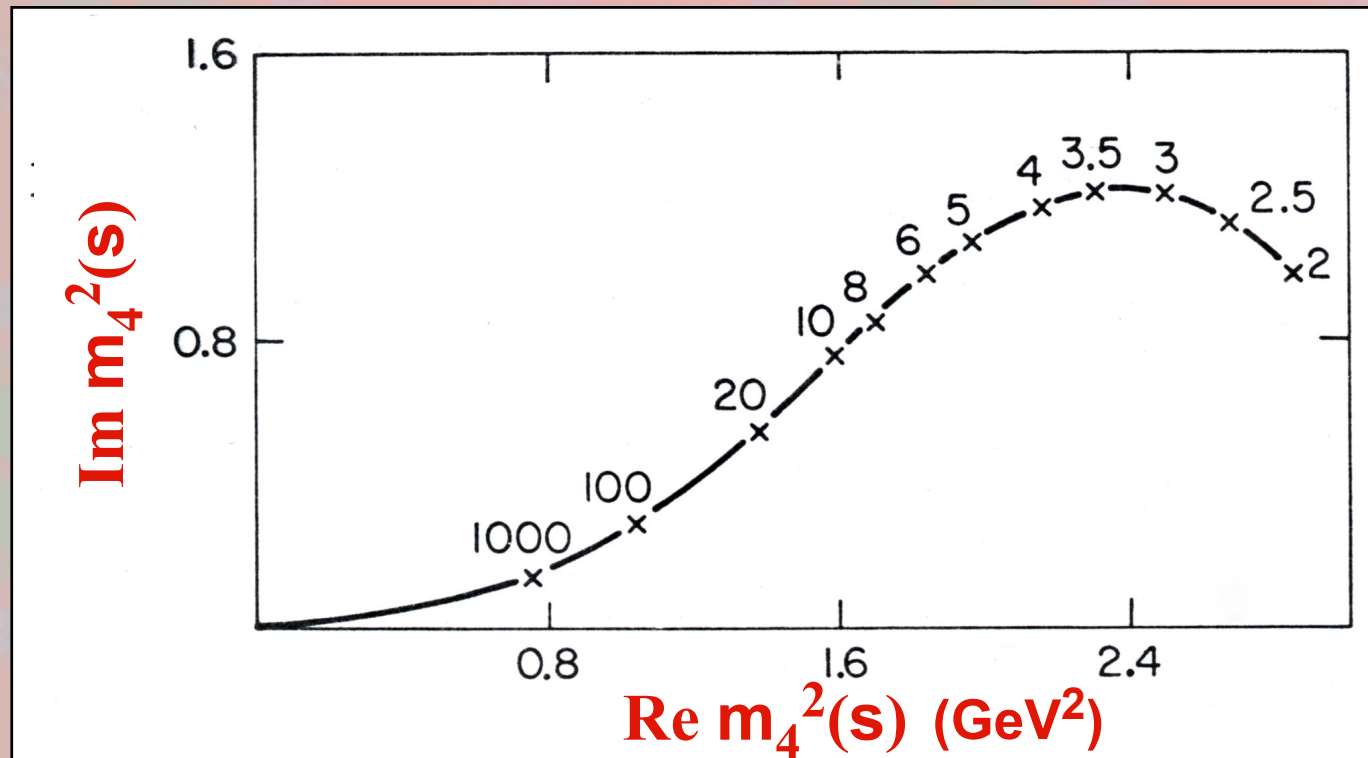
$$\frac{1}{\alpha(s, m^2)} = \frac{1}{\alpha(\mu^2, m^2)} + \frac{1}{4\pi} \left[ 11 \ln \frac{s}{\mu^2} - \frac{2}{3} \sum_j \int_{\mu^2}^s \frac{dz}{z} F_1 \left( \frac{m_j^2}{z} \right) \right]$$

$$F_1(x) = 1 - 6x + \frac{12x^2}{\sqrt{1+4x}} \ln \left[ \frac{\sqrt{1+4x} + 1}{\sqrt{1+4x} - 1} \right]$$

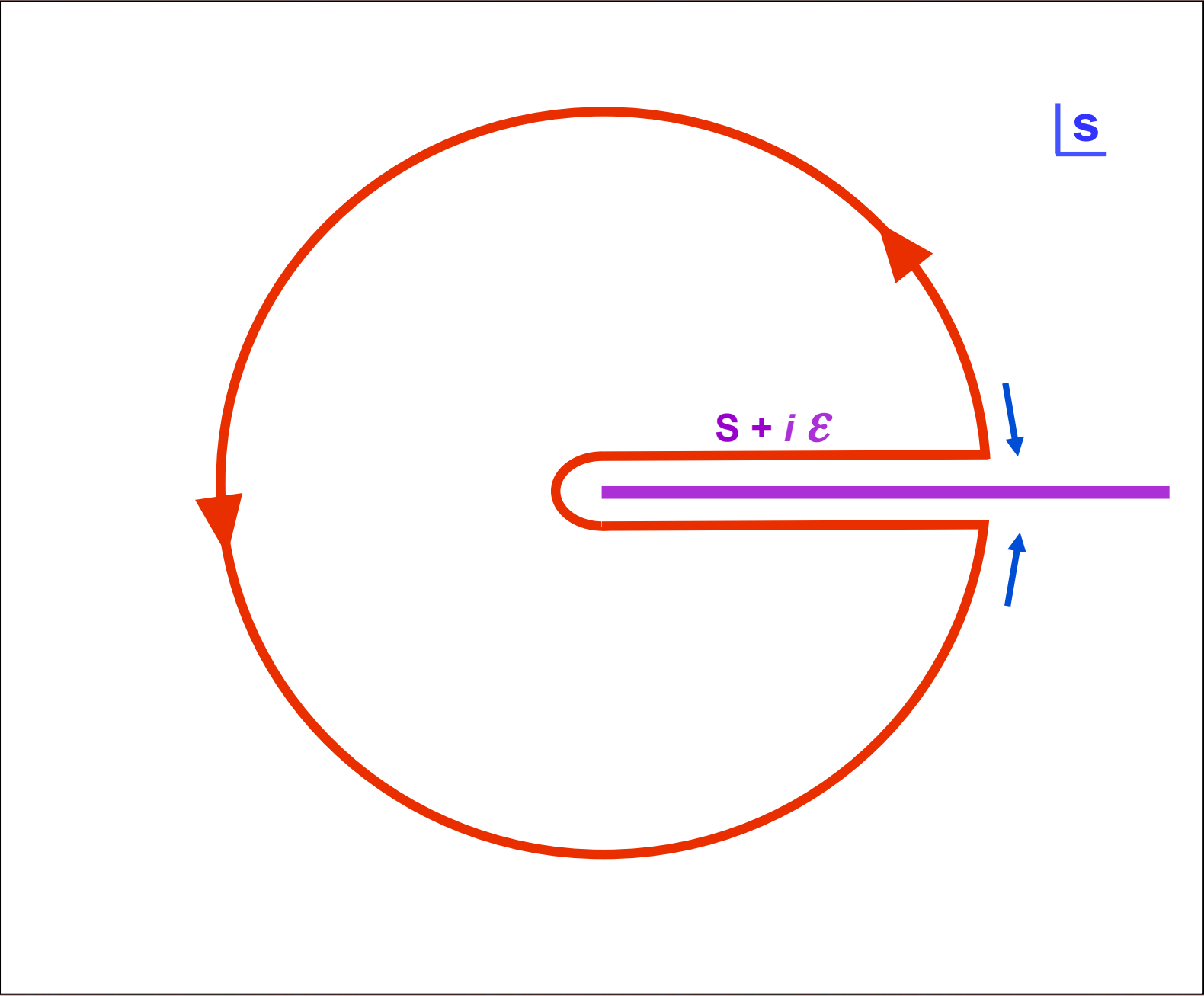


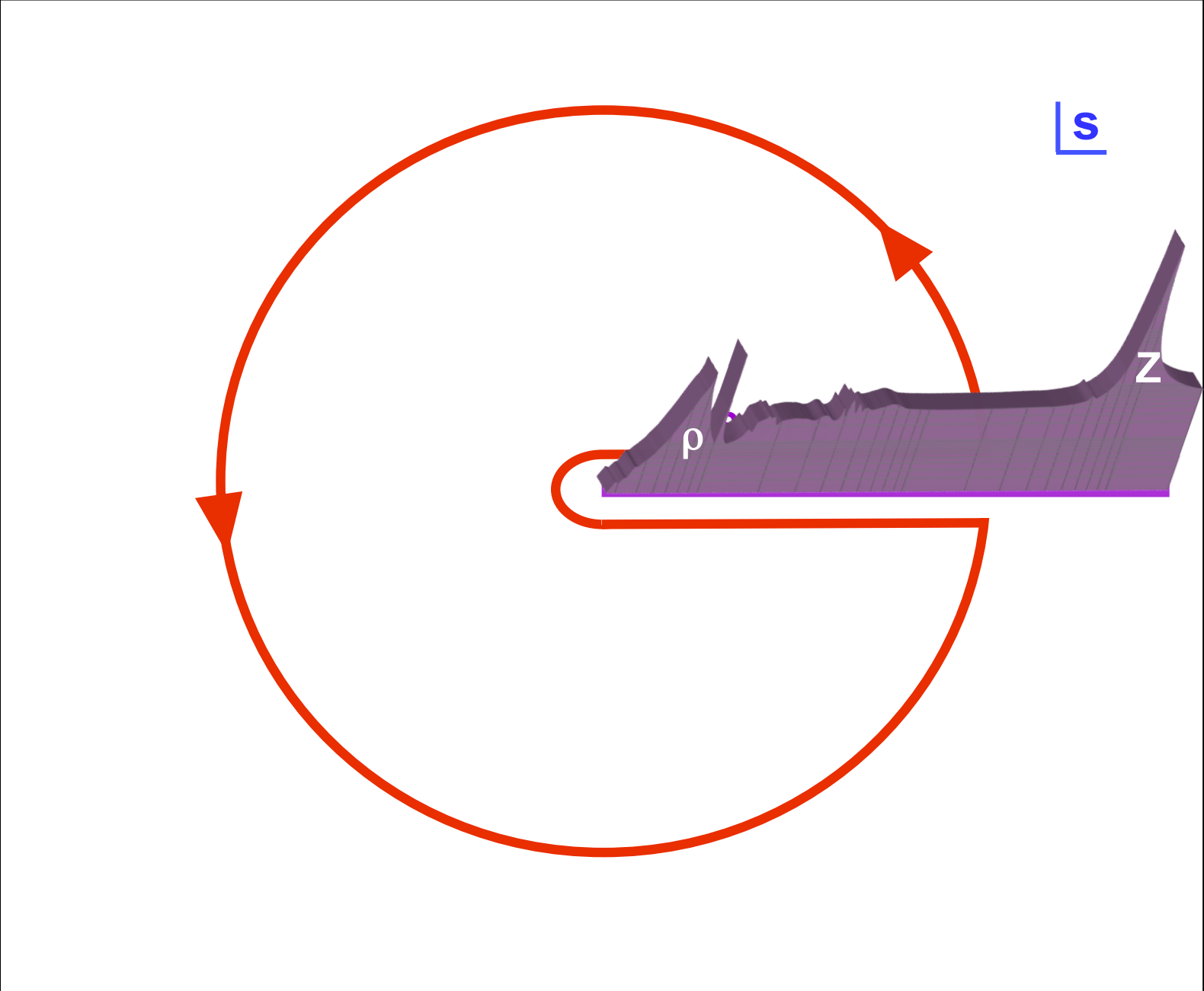
$$S_F^{-1}(p, m(\mu^2)) \Big|_{p^2=\mu^2} = \not{p} - m(\mu^2) \quad \text{defining mass at renorm. pt}$$

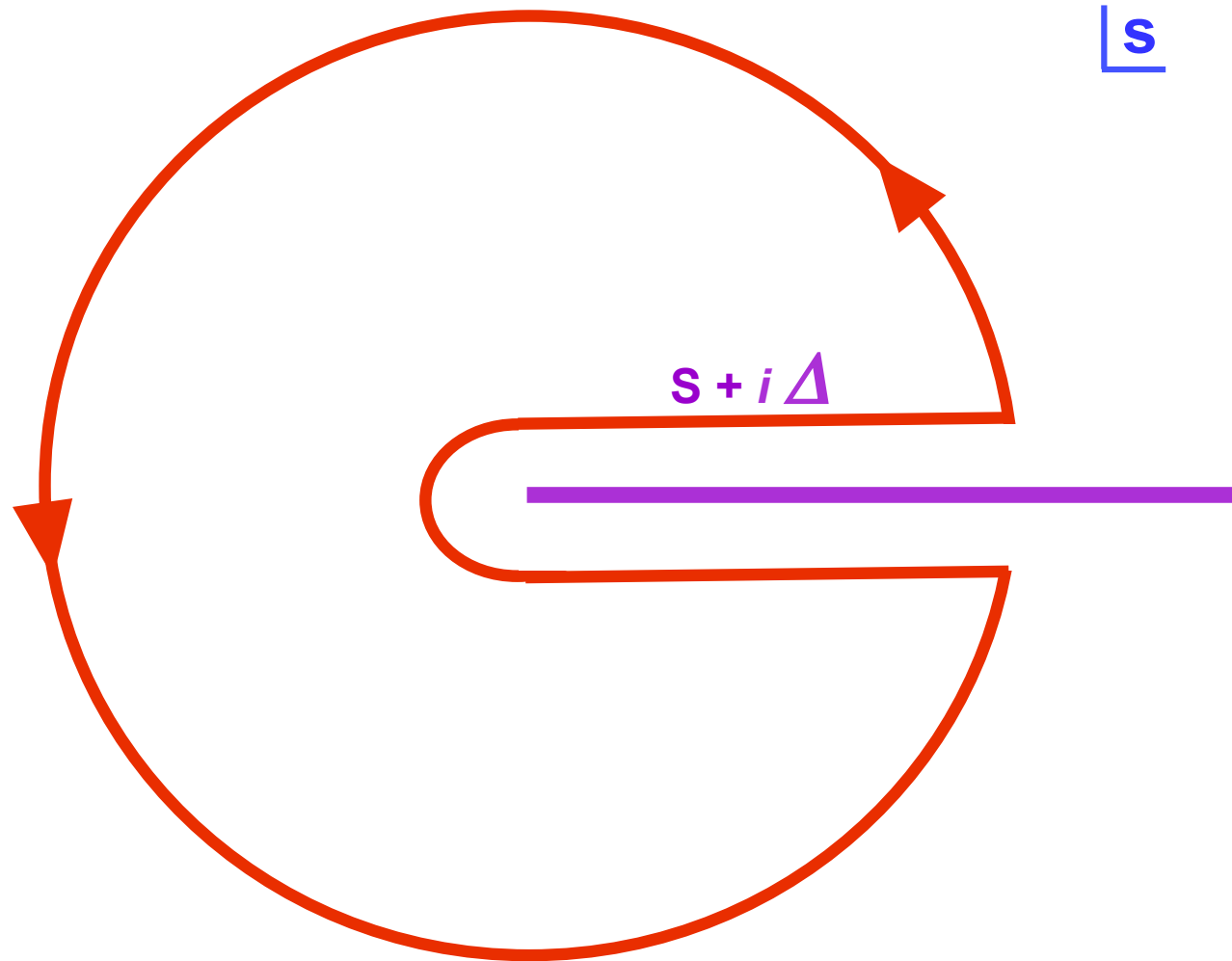
$$m^2(s) \simeq m^2(\mu^2) \left( \frac{\alpha(s)}{\alpha(\mu^2)} \right)^{d_m}$$









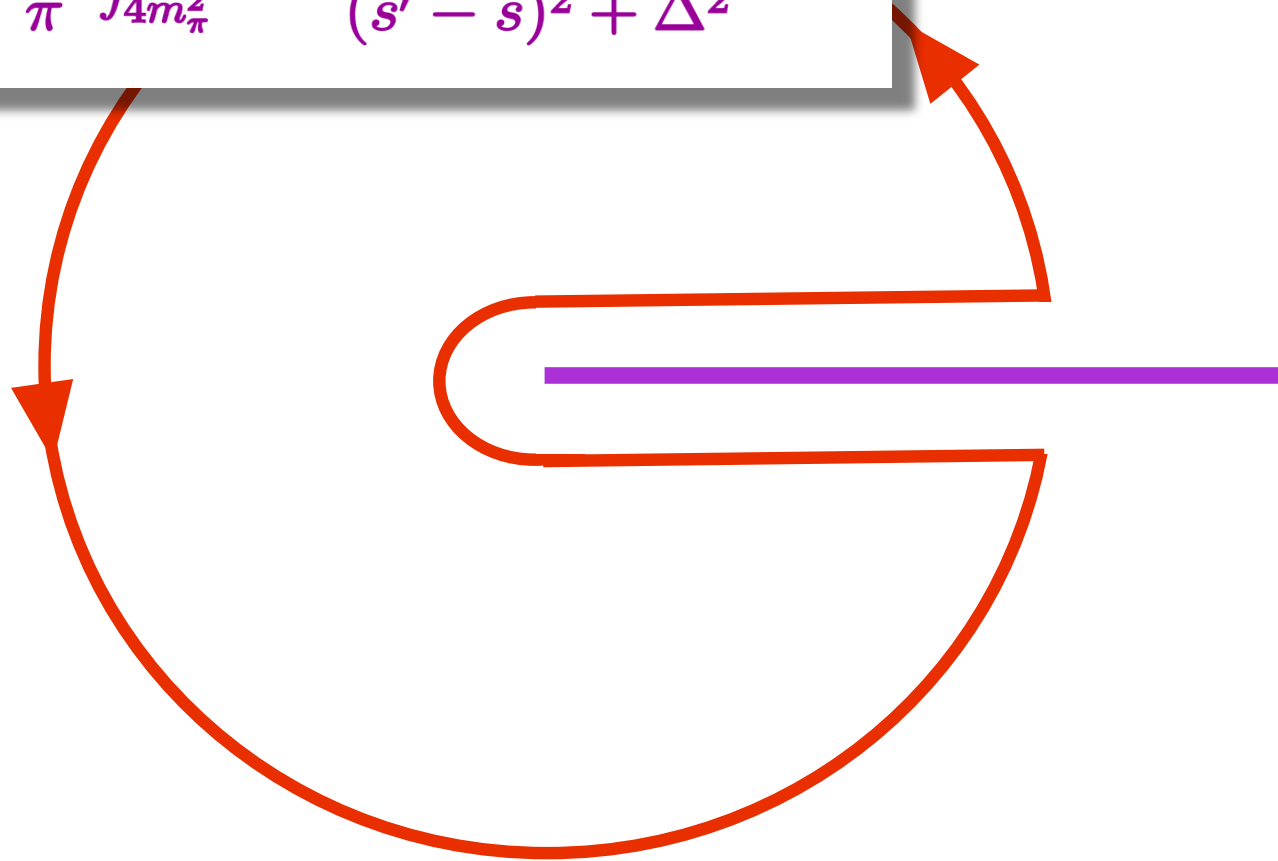


Poggio, Quinn, Weinberg

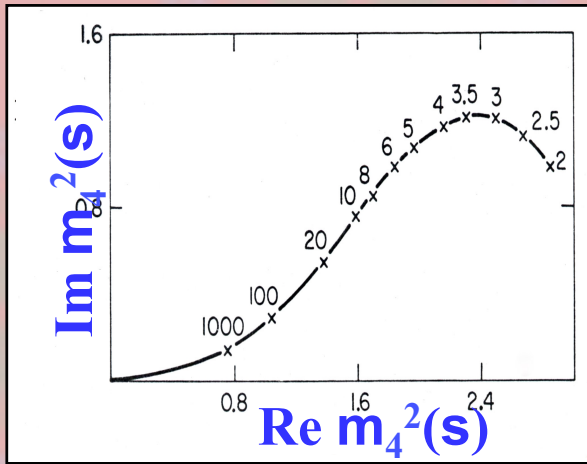
$$\mathcal{R}(s, \Delta) = \frac{1}{2i} [\Pi(s + i\Delta) - \Pi(s - \Delta)]$$

$$= \frac{\Delta}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{R(s')}{(s' - s)^2 + \Delta^2}$$

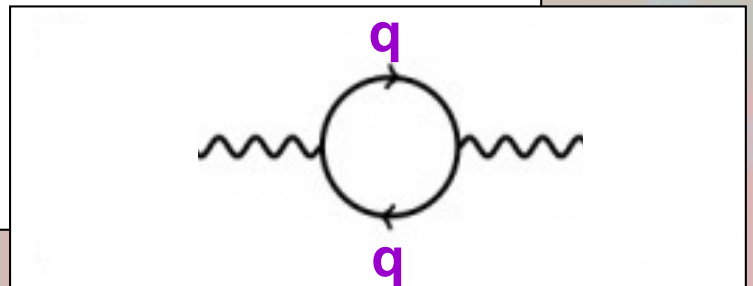
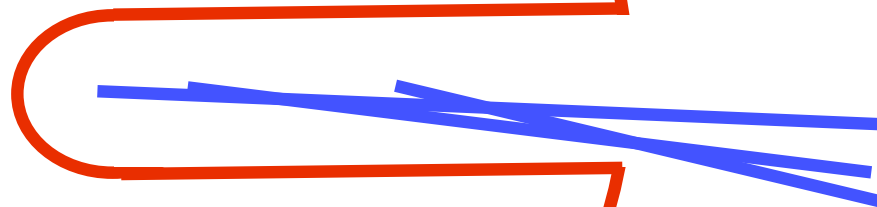
$\lfloor s$

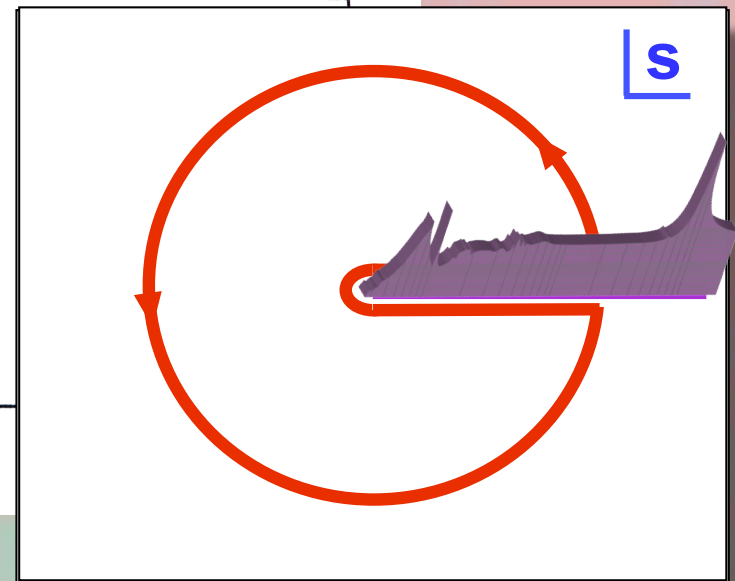
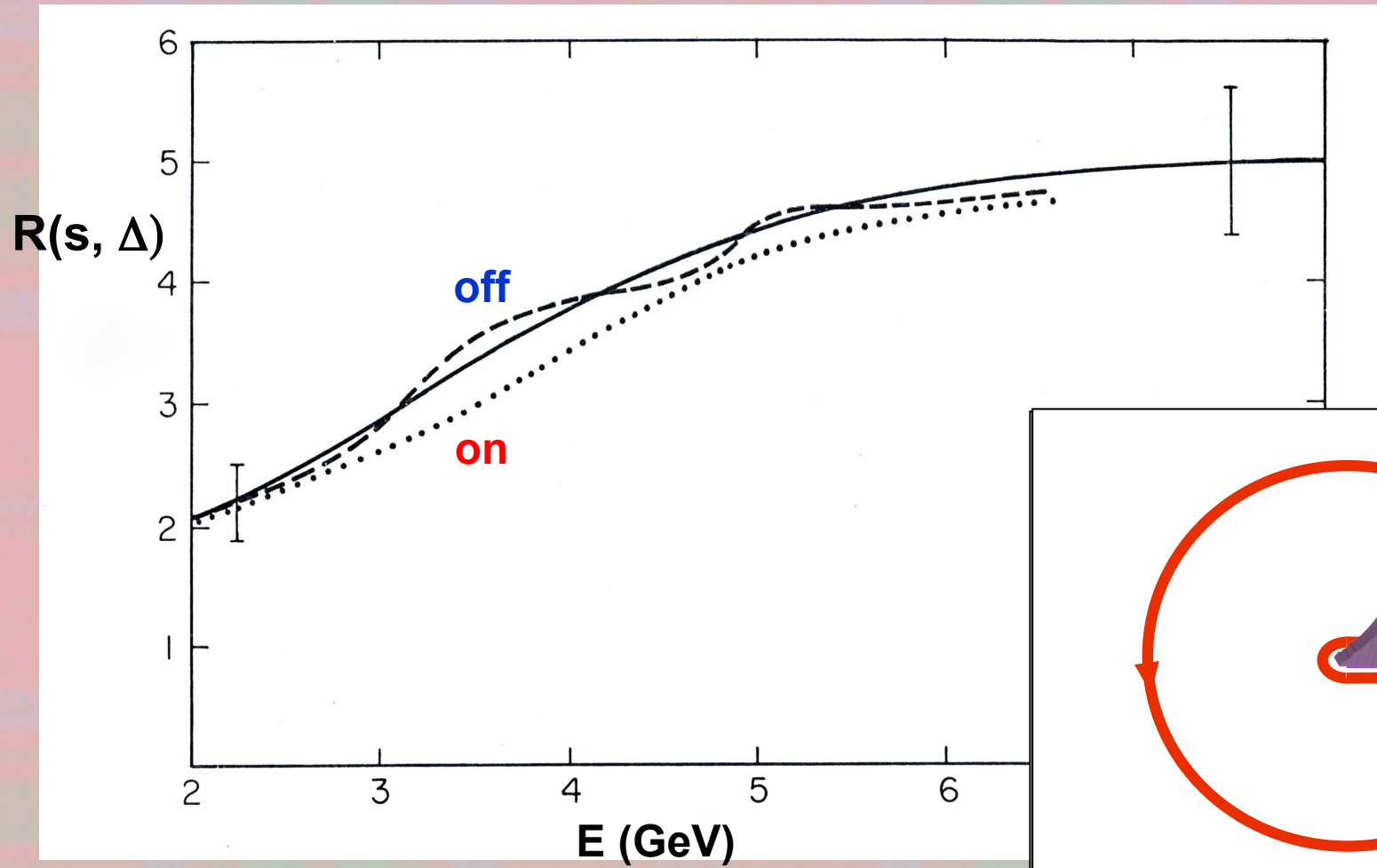


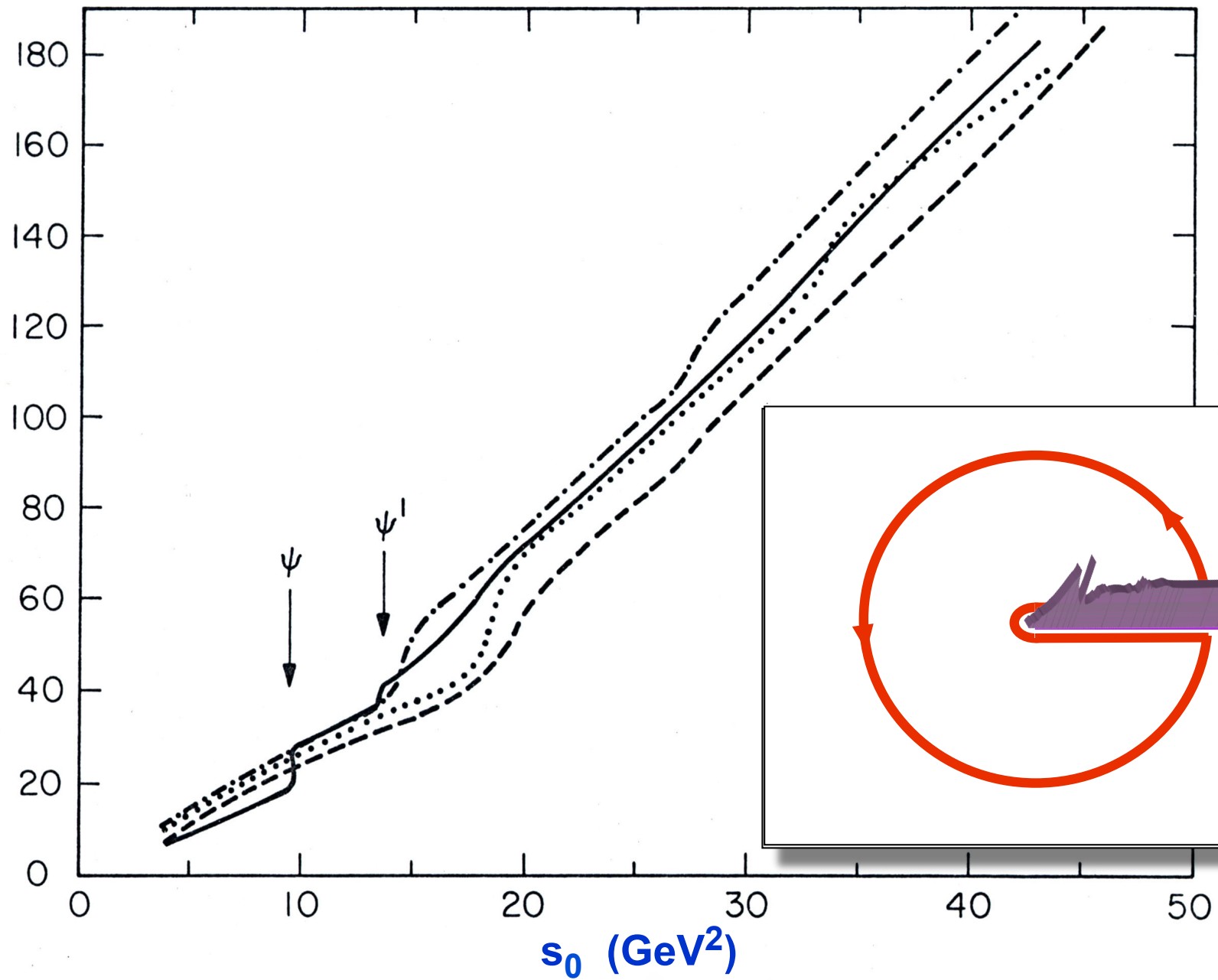
Poggio, Quinn, Weinberg

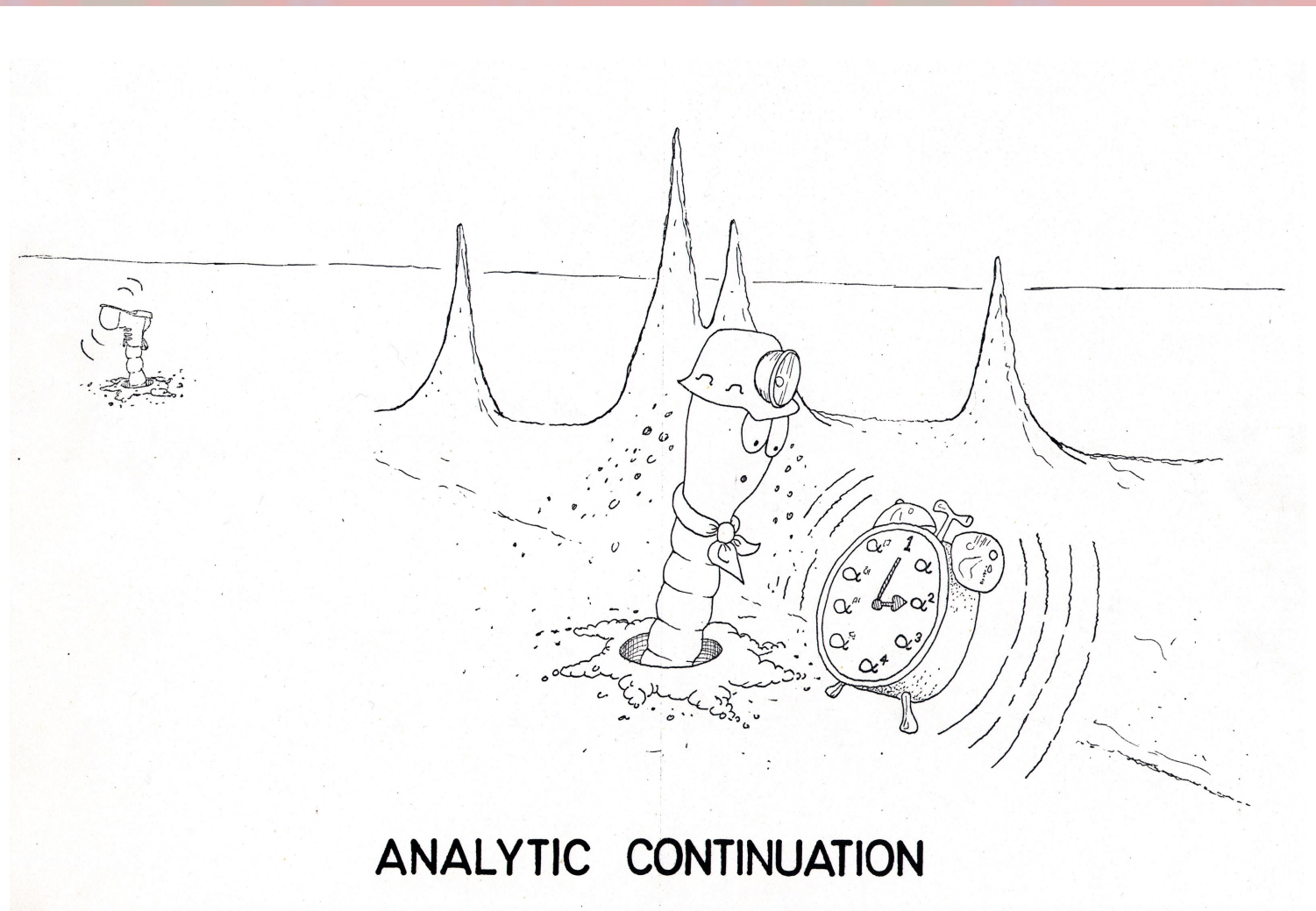


$s$





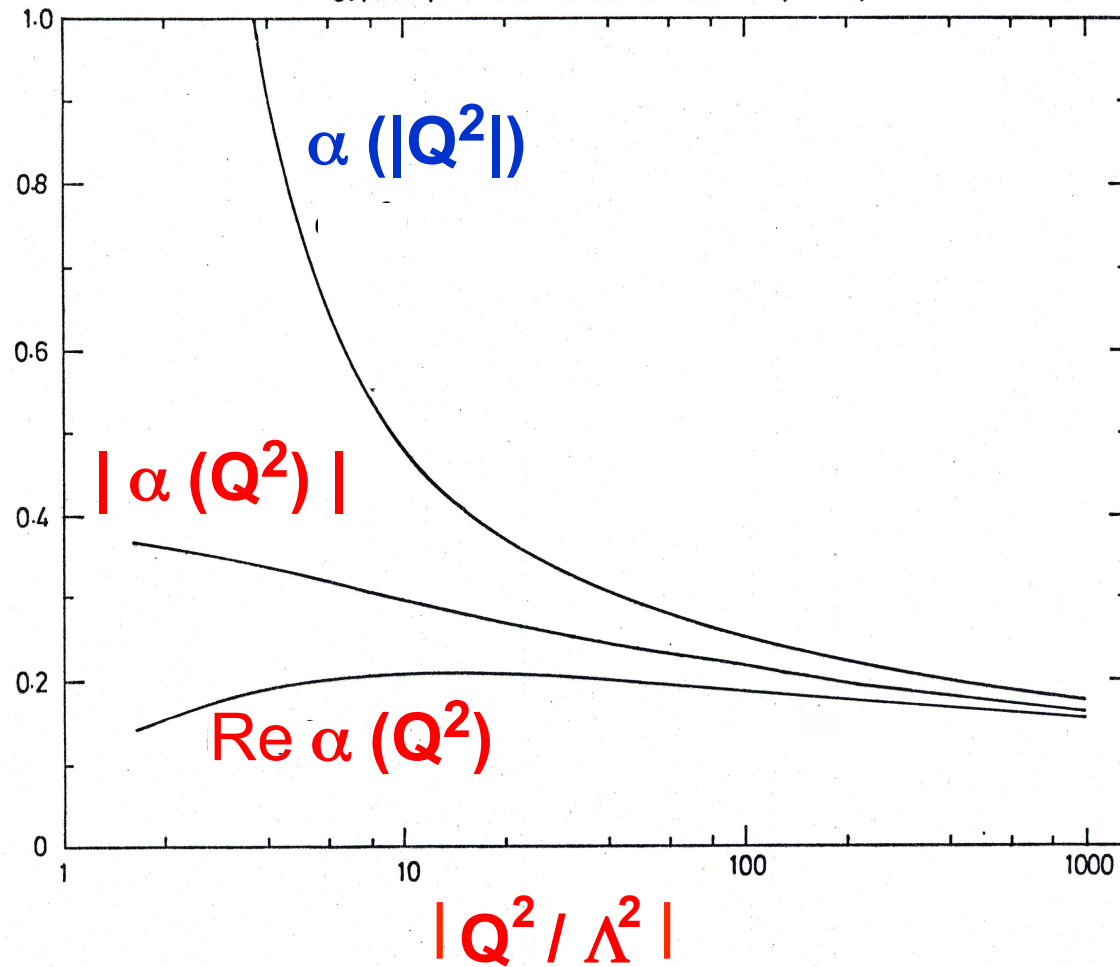




**ANALYTIC CONTINUATION**



$\alpha ( Q^2 / \Lambda^2 )$  in timelike region  $Q^2 < 0$



$$A \alpha(s) + B \pi^2 \alpha^3(s) + \dots \longrightarrow A a(s)$$



PHYSICAL REVIEW D  
VOLUME 33, NUMBER 3  
1 FEBRUARY 1986  
Total  $e^+e^-$  annihilation cross section in the charm continuum  
E. Schuler\*  
Department of Physics, University of California, Los Angeles, California 90024  
Received 10 November 1985

PHYSICS REPORTS (Review Section of Physics Letters) 127, No. 1 (1985) 1-97, North-Holland, Amsterdam  
**HADRON PROPERTIES FROM QCD SUM RULES**  
L.J. REINDERS\*, H. RUBINSTEIN\*\* and S. YAZAKI\*\*\*

PHYSICAL REVIEW D  
VOLUME 18, NUMBER 8  
15 OCTOBER 1978  
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Thomas Appelquist and R. Shankar  
Department of Physics, Yale University, New Haven, Connecticut 06516  
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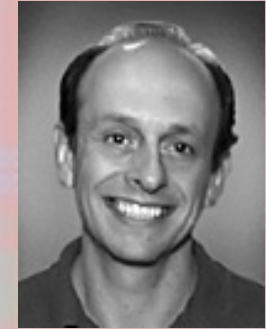
PHYSICS LETTERS  
Volume 81B, number 2  
12 February 1979  
QUANTUM CHROMODYNAMICS AND THE DECAY OF THE  $\tau$  LEPTON  
Otto NACHTMANN and Werner WETZEL  
Institut für Theoretische Physik der Universität Heidelberg, Heidelberg, Germany  
Received 10 November 1978

Nuclear Physics B135 (1978) 66-92  
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**ASPECTS OF THE GRAND UNIFICATION OF STRONG, WEAK AND ELECTROMAGNETIC INTERACTIONS**  
A.J. BURAS\*, J. ELLIS, M.K. GAILLARD\*\* and D.V. NANOPOULOS\*\*\*  
CERN, Geneva, Switzerland  
Received 10 November 1978

PHYSICS LETTERS B  
19 November 1978  
Physics Letters B 440 (1978) 367-374  
Constraints on hadronic spectral functions from continuous families of finite energy sum rules  
Kim Maltman<sup>1</sup>  
Department of Mathematics and Statistics, York University, 4700 Keele St., Toronto, Ont. M3J 1P3, Canada  
Received 10 November 1978

Nuclear Physics B179 (1981) 171-188  
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**RENORMALIZATION GROUP ESTIMATE OF THE HADRONIC DECAY WIDTH OF THE HIGGS BOSON**  
Takeo INAMI  
Institute of Physics, University of Tokyo, Komaba, Meguro-ku, Tokyo, Japan 153  
Takahiro KUBOTA<sup>1</sup>  
Department of Physics, University of Tokyo, Tokyo, Japan 113  
Received 10 November 1980

Nuclear Physics B146 (1978) 283-284  
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**AMBIGUITIES FROM QUARK MASSES IN THE RENORMALIZATION GROUP\***  
H. David POLITZER  
California Institute of Technology, Pasadena, California 91125, USA  
Received 14 August 1978



## AMBIGUITIES FROM QUARK MASSES IN THE RENORMALIZATION GROUP \*

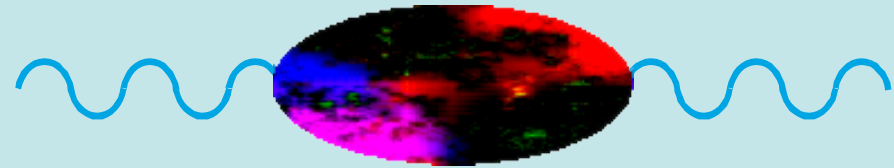
H. David POLITZER

Renormalizability ensures that any consistent prescription will lead to the same physical predictions, whether the  $\beta$  functions are the same or not. More precisely, any discrepancies between two calculations carried out to a given order must be yet higher order in the coupling constant. One may still ask, in the spirit of Moorhouse, Pennington and Ross [4], whether one particular prescription is better than others in the following practical sense: if we compute to lowest order and ignore yet higher orders, may one prescription be closer to the complete theory than another? That is to ask: can choice of a particular prescription minimize the numerical coefficient of  $g^2$  in the next correction? Typically the answer is yes, but it is impossible to prove without actually computing that next correction. However, for the bulk of phenomenological applications, the use of the light quark-gluon vertex to define  $g$  seems a likely candidate because it is precisely that vertex which occurs in lowest-order amplitudes and is subsequently renormalized by higher orders.

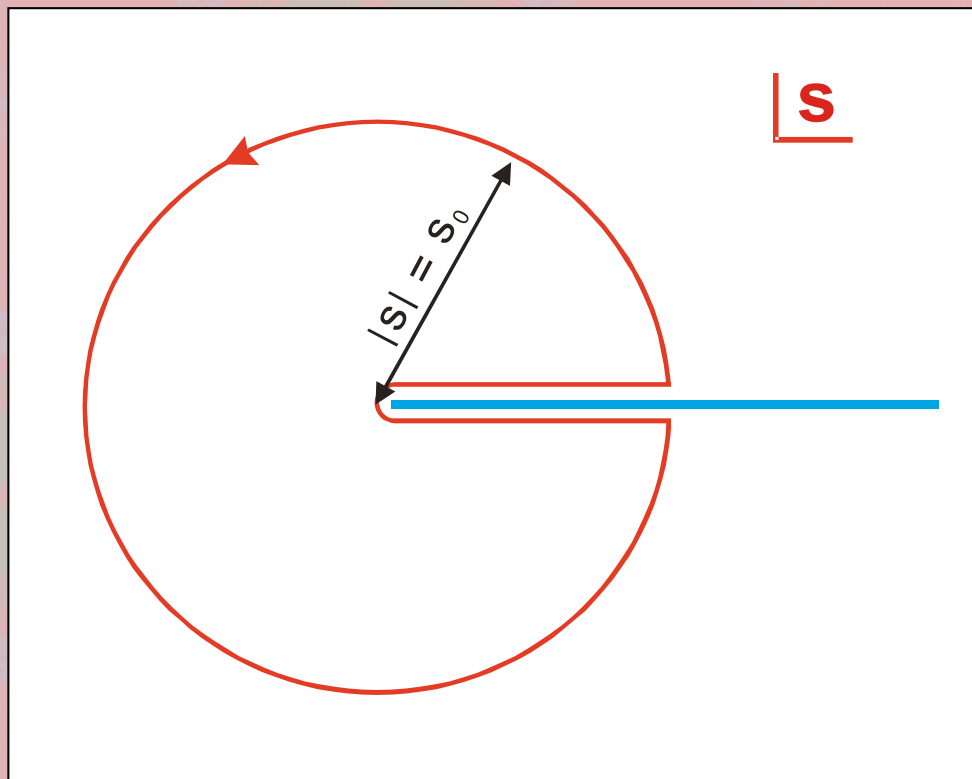
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E.C. Poggio, H.R. Quinn and S. Weinberg, Phys. Rev. D13 (1976) 1958;  
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- [4] R.G. Moorhouse, M.R. Pennington and G.G. Ross, Nucl. Phys. B124 (1977) 285.

# QCD Sum Rules

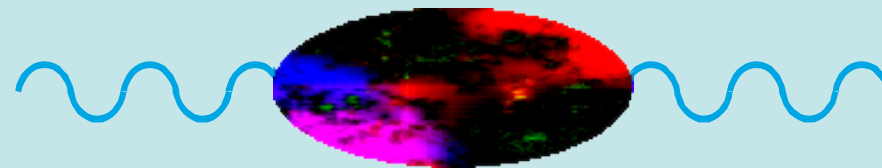


current correlator

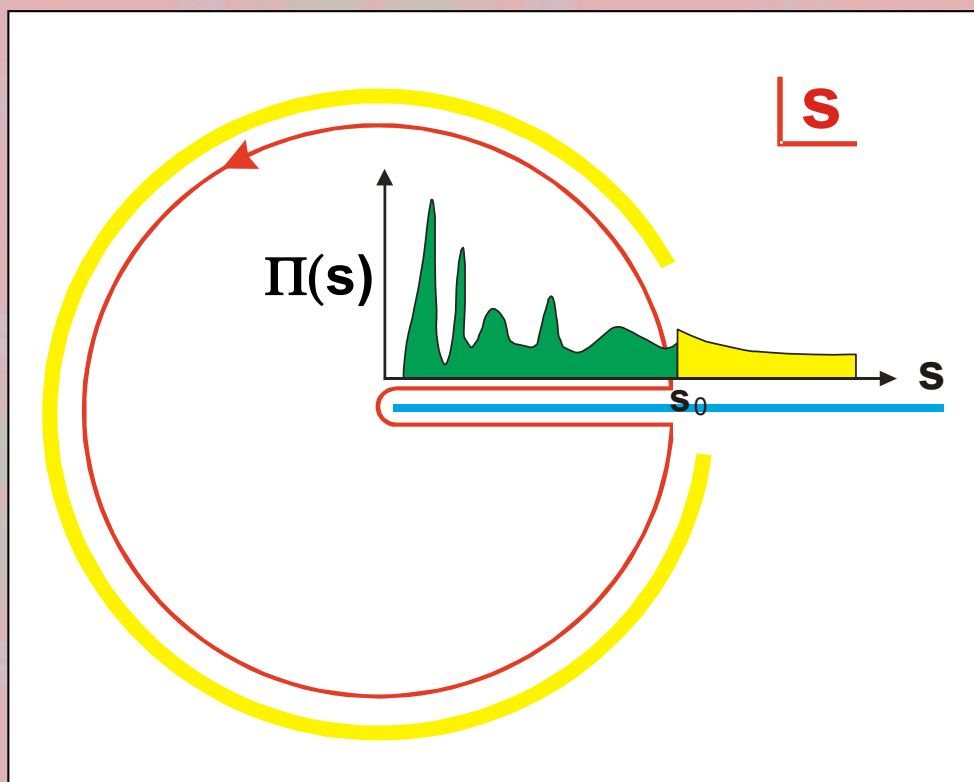


$$\oint ds \omega(s) \Pi(s) = 0$$

# QCD Sum Rules

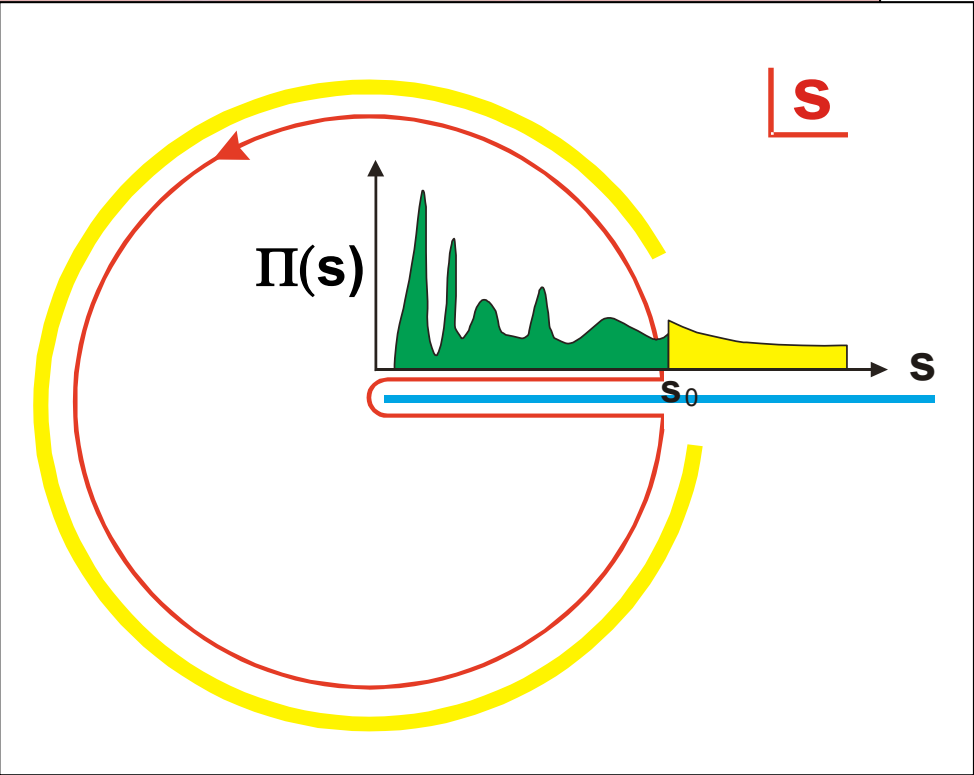
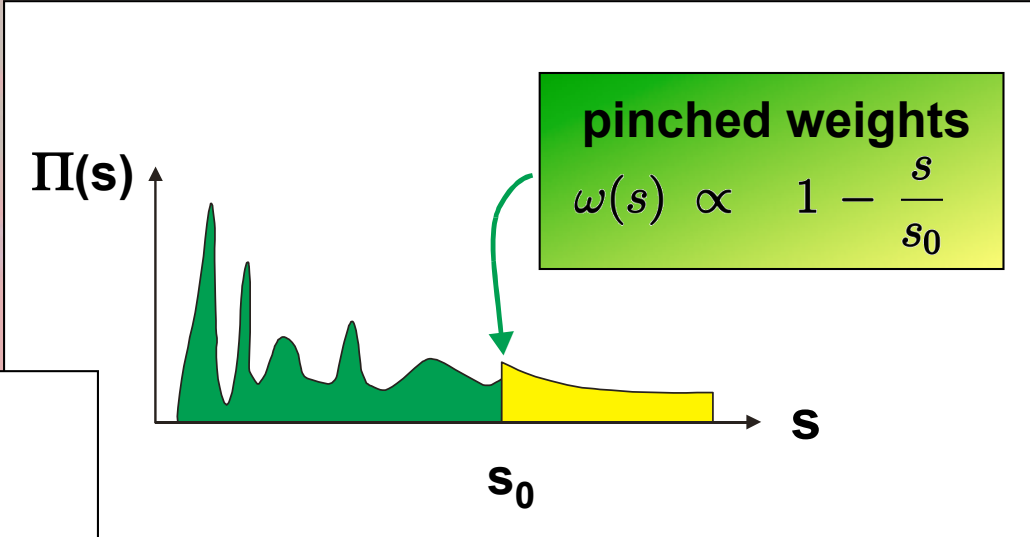


current correlator



$$\langle qq \rangle_0, \langle \alpha GG \rangle_0, \dots$$

$$2i \int_0^{s_0} ds \omega(s) \operatorname{Im} \Pi(s) = - \oint_C ds \omega(s) \Pi(s)$$



# working with **Graham Ross** 1974-1984

